

# The integer programming extreme value (IPEV) model:

An application for estimation of the leisure trip demand

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**Abstract:**

We developed an integer programming extreme value (IPEV) model that accounts the integer property of trip data and has the same advantages as the multiple discrete–continuous extreme value choice (MDCEV) model. The proposed model is consistent with utility theory and provides a single structural framework for simultaneously modeling the choice of alternatives and quantity decisions with the constraint of the integer value of consumption. We demonstrate that the proposed model has a closed-form probability expression. Finally, we apply the proposed model to the recreation demand for national parks in Japan. The empirical results suggest that the proposed model provides a better fit for the data than the previous model and that ignoring the integer property of demand might cause an underestimation of the welfare loss.

**Keywords:**

demand analysis, utility theory, integer demand, MDCEV, leisure trip demand

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## 1. Introduction

Standard models of consumer demand assume continuity of consumption and an interior solution to the consumer's utility maximization problem. However, these assumptions are unrealistic in many settings. For example, leisure trip demand exhibits the characteristics of count data; that is, the number of trips is always a non-negative integer value. Moreover, for a given individual, some sites are visited once or more (interior solutions), while others are not visited (corner solutions). To analyze such data on the integer demand of multiple alternatives, econometric models that account for a mixture of corner and interior solutions with the constraint of non-negative integer demand are required. Similar corner solutions with integer demand emerge in transportation mode (e.g., whether to use a car or bus and how many times to ride), vehicle demand (e.g., which type of and how many vehicles to buy), food demand (e.g., which brand of cereal to buy and how many boxes), and labor demand (e.g., whether to employ part-time or full-time workers and how many workers to employ).

Under the assumption of continuous demand, the multiple discrete–continuous extreme value choice (MDCEV) model (Bhat, 2005, 2008) was proposed to analyze the demand for multiple alternatives with the corner solution. The MDCEV model provides a single structural framework that simultaneously models the consumer choice situation and purchase quantity decisions for multiple alternatives, with the possibility of a corner solution. Furthermore, the MDCEV model has a closed-form probability of observing demand; it is a general model that includes the logit model as a special case. However, the MDCEV model uses the Karush-Kuhn-Tucker (KKT) condition to estimate the parameters. Therefore, the MDCEV model requires the assumption of continuous demand, which may be unrealistic for leisure trips or vehicle choices.

In this study, we propose an integer programming extreme value (IPEV) model, which is an integer demand version of the MDCEV model. The proposed IPEV model, which applies the integer programming method to the MDCEV model, accounts for non-negative integer demand with multiple alternatives. Our proposed model is appealing in that it satisfies the constraint imposed by the non-negative integer amount of consumption, in addition to having the same advantages as the MDCEV continuous demand approach. The integer programming problem is known to be nondeterministic polynomial-time hard (NP-hard), and finding an exact solution to the problem might be difficult (Wolsey, 1998; Wolsey and Nemhauser, 1999; Vazirani, 2001). In the integer and combinatorial optimization literature, approximate algorithms, including local search and greedy methods, have been proposed (Sait and Youssef, 2000; Aarts and Lenstra, 2003). This study investigates the implementation of approximate algorithms to the proposed IPEV model for the estimation of parameters, demand prediction, and welfare analysis. In the empirical section, we consider the application of the IPEV model to the leisure trip demand of national parks in Japan and compare the results of our proposed model with those of the continuous demand MDCEV model.

The contribution of this study can be summarized as follows:

- This study develops an econometric IPEV model that does not require the KKT condition used by the MDCEV model to estimate the parameters. Instead, the proposed IPEV model uses a local search algorithm to determine the optimization condition with the constraint of a non-negative integer demand.
- We show that the proposed IPEV model has the same advantages as the MDCEV model. The IPEV model can analyze the demand for multiple alternatives with corner and interior solutions; it has a closed-form probability of observing demand, and it is a general model that includes the conditional logit model as a special case.
- This study proposes an integer programming approach for demand prediction and welfare analysis of integer demand.
- The empirical results suggest that our proposed IPEV model provides a better fit for leisure trip data than the MDCEV model. Furthermore, we show that ignoring the integer property of demand may cause an underestimation of welfare loss.

The remainder of this paper is organized as follows. Section 2 reviews the literature. Section 3 provides an overview of both the continuous demand model and the proposed integer programming model. Section 4 presents the proposed strategy for demand prediction and welfare analysis using integer-programming algorithms. Section 5 presents the results of our empirical analysis of leisure trip data from the national parks in Japan. Finally, Section 6 provides the concluding remarks.

## 2. Related works

Several approaches have been proposed for demand analysis of multiple goods using corner solutions. The first approach is the multivariate count data models (Zhang et al., 2017), which are the multivariate version of the count data models of a single good, including Poisson and negative binomial regression models (Cameron and Trivedi, 2013). Multivariate count data models have the advantage of a closed-form probability expression for estimating integer demand for multiple goods. However, these multivariate count models are not based on underlying utility theory. Therefore, these models may not be suitable for economic welfare analyses, although a theoretical foundation for count data models has been developed (Hellerstein and Mendelsohn, 1993).

The second approach is the linked model, which segments the consumer's decision into two components: the choice of alternative and quantity decisions (Bockstael et al., 1986). This model combines the estimation results of the discrete choice and count regression models. Therefore, the linked model can

handle the properties of an integer value of consumption. However, this model is not derived from the unified utility theory.<sup>1</sup>

The third approach is the Kuhn-Tucker (KT) model,<sup>2</sup> which uses the KKT condition for utility maximization to estimate the parameters of the utility function (von Haefen and Phaneuf, 2005). The KT model is based on utility maximization behavior. The KT model has the advantage of providing a single structural framework to simultaneously model alternative selection and quantity decisions using a mixture of interior and corner solutions. However, the KT model does not have a closed-form probability expression. It is necessary for the KT model to numerically calculate the likelihood. Furthermore, the KT model assumes continuous demand and is inconsistent with discrete choice models. To address the integer demand for the KT model, an integer-programing approach was proposed (Kuriyama and Hanemann, 2006; Lee and Allenby, 2014). Nevertheless, as Bhat (2022) shows, the integer programming approach to the KT model has no closed-form probability expressions. Furthermore, determining the relationship between integer-programing KT and discrete choice models is difficult.

The fourth approach is the MDCEV model (Bhat, 2005, 2008) that also uses the KKT condition to estimate the utility function parameters. Therefore, the MDCEV is consistent with the utility theory, similar to the KT model. The difference between the KT and MDCEV models lies in the assumption that the outside good is deterministic or random (Bhat, 2008). For the KT model, the utility of the outside good (the Hicks composite good) is assumed to be deterministic. For the MDCEV model, the utility of all goods, including outside good, is assumed to have error components with an extreme-value distribution. The MDCEV model has the advantage of having a closed-form probability expression for the observed demand. Furthermore, as Bhat (2008) shows, the MDCEV is a general model that includes a conditional logit model and a KT model for special cases. However, the MDCEV model assumes continuous demand. Therefore, the estimation results of the integer demand using the MDCEV model may be biased. Recently, Bhat (2022) proposed the multiple discrete-count extreme value (MDCNTEV) model, which is a utility-theoretic model for multiple discrete-count data that links MDCEV and count models. The MDCNTEV model can deal with the property of integer demand of the total consumption. Nevertheless, it has the MDCEV component that requires the assumption of a continuous fractional split in the total demand.

In contrast, this study proposes the IPEV model, which is an integer-programing approach to the MDCEV model. It has the same advantages as the MDCEV model. This is a utility-theoretic econometric model for multiple alternatives with a mixture of interior and corner solutions. It has a closed-form expression for the probability of observing consumption. It is a general model that includes the conditional

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<sup>1</sup> Hausman et al. (1995) claimed that the linked model is utility-consistent for the utility function of Gorman generalized polar foam. However, Smith (1997) showed that their proof of consistency relies upon an assumption that holds only under restrictive circumstances. See also Herriges et al. (1999).

<sup>2</sup> The KT demand model was proposed by Hanemann (1978) and Wales and Woodland (1983). Phaneuf et al. (2000) provided an empirical application of this model to estimate recreation demand.

logit model for special cases. Moreover, the proposed IPEV model is consistent with non-negative integer demand.

### 3. Model

#### 3.1 Conceptual model

Figure 1 provides the conceptual models of the continuous and integer demands for two goods,  $x = (x_1, x_2)$ . Let  $U(x)$  be the utility function and  $x_1 > 0$  be the numeraire or outside good with price  $p_1 = 1$ . First, we consider the case of continuous demand. Let  $x_2$  be a continuous consumption good with price  $p_2$ . The dotted line of AD is the budget constraint. Let  $x^* = (x_1^*, x_2^*)$  be the optimal consumption set for continuous demand. Panel (a) shows the interior solution ( $x_2^* > 0$ ). For the interior solution, the optimal point is E, and the optimal condition is  $p_2 \partial U / \partial x_1 = \partial U / \partial x_2$ . Panel (b) shows the corner solution ( $x_2^* = 0$ ). For this case, the optimal point is A, and the optimal condition is  $p_2 \partial U / \partial x_1 > \partial U / \partial x_2$ .

Figure 1. Continuous and integer demand models

Subsequently, consider the case of integer demand such that the consumption of  $x_2$  is constrained to a non-negative integer value. Let  $\tilde{x} = (\tilde{x}_1, \tilde{x}_2)$  be the consumption set of the integer demand. Under the budget constraint, the possible consumption of  $x_2$  is  $\tilde{x}_2 \in \{0, 1, 2, 3\}$ . Let  $\tilde{x}^* = (\tilde{x}_1^*, \tilde{x}_2^*)$  be the optimal consumption set for the integer demand. Panel (a) shows the interior solution ( $\tilde{x}_2^* > 0$ ). For the interior solution, the optimal point of integer demand is C, and the optimal consumption of Good 2 is  $\tilde{x}_2^* = 2$ . Consider the case of increasing or decreasing the consumption of Good 2 by one unit. Let  $\tilde{x}^+ = (\tilde{x}_1^* - p_2, \tilde{x}_2^* + 1)$  be the consumption set of D and  $\tilde{x}^- = (\tilde{x}_1^* + p_2, \tilde{x}_2^* - 1)$  be the consumption set of B. The utility of the optimal point C must be higher than that of points D and B. Thus, the optimal condition of the interior solution for integer demand requires  $U(\tilde{x}^*) > U(\tilde{x}^+)$  and  $U(\tilde{x}^*) > U(\tilde{x}^-)$ . Panel (b) shows the corner solution ( $\tilde{x}_2^* = 0$ ). In this case, the optimal point is A, and the utility of optimal point A must be higher than that of point B. Thus, the optimal condition of the corner solution for integer demand is  $U(\tilde{x}^*) > U(\tilde{x}^+)$ .

#### 3.2 The continuous demand MDCEV model

This section summarizes the continuous demand MDCEV model proposed by Bhat (2008). Consider a generalized variant of the translated CES utility function with an outside good:

$$U(x) = \sum_{k=1}^K U_k(x_k) = \frac{\psi_1}{\alpha_1} x_1^{\alpha_1} + \sum_{k=2}^K \frac{\gamma_k \psi_k}{\alpha_k} \left\{ \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\}, \quad (1)$$

$$\begin{aligned}\psi_1 &= \exp(\varepsilon_1), \\ \psi_k &= \exp(\beta' z_k + \varepsilon_k),\end{aligned}$$

where  $U(x)$  is a quasi-concave, increasing, and continuously differentiable function with respect to the consumption goods  $x = (x_1, \dots, x_K)$ ;  $x_1$  is the numeraire or an outside good that is always consumed in positive quantities;  $z_k$  is a vector of attributes of good  $k$  and the individual attributes;  $\varepsilon_k$  is a vector of random components that are assumed to be known to the individual but unknown to the analyst; and  $\alpha_k$ ,  $\beta$ , and  $\gamma_k$  are the estimated parameters. The individual is assumed to maximize the utility function, subject to the budget constraint. The Lagrangian of this utility maximization problem is as follows:

$$\mathcal{L} = \frac{\psi_1}{\alpha_1} x_1^{\alpha_1} + \sum_{k=2}^K \frac{\gamma_k \psi_k}{\alpha_k} \left\{ \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\} + \lambda \left( E - \sum_{k=1}^K p_k x_k \right), \quad (2)$$

where  $E$  is income,  $p_k$  is the price of good  $k$ , and  $\lambda$  is the Lagrangian multiplier. The KKT condition for this problem is given as

$$\begin{aligned}\psi_k \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} &= \lambda p_k, \text{ if } x_k^* > 0, \\ \psi_k \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} &< \lambda p_k, \text{ if } x_k^* = 0,\end{aligned} \quad (3)$$

where  $x_k^*$  is the solution to the optimization problem. The first good is assumed to be the outside good  $x_1^* > 0$ ; therefore, the KKT condition can be rewritten as

$$\begin{aligned}V_k + \varepsilon_k &= V_1 + \varepsilon_1, \text{ if } x_k^* > 0, \\ V_k + \varepsilon_k &< V_1 + \varepsilon_1, \text{ if } x_k^* = 0,\end{aligned} \quad (4)$$

where  $V_1 = (\alpha_1 - 1) \ln x_1 - \ln p_1$  and  $V_k = \beta' z_k + (\alpha_k - 1) \ln \left( \frac{x_k^*}{\gamma_k} + 1 \right) - \ln p_k$ . Consider an individual who chooses to consume only the first  $M$  of  $K$  goods; thus,  $x_k^* > 0$  for  $k = 1, \dots, M$  and  $x_k^* = 0$  for  $k = M + 1, \dots, K$ . If the  $\varepsilon_k$ 's are independent and identically distributed draws from the type-I extreme value distribution with inverse scale parameter  $\sigma$  for all  $k$ , Bhat (2008) shows that the probability of observing  $x^* = (x_1^*, \dots, x_K^*)$  has closed form:

$$P(x^*) = \frac{1}{p_1} \frac{1}{\sigma^{M-1}} \left[ \prod_{i=1}^M f_i \right] \left[ \sum_{i=1}^M \frac{p_i}{f_i} \right] \left[ \frac{\prod_{i=1}^M e^{V_i/\sigma}}{(\sum_{k=1}^K e^{V_k/\sigma})^M} \right] (M-1)!, \quad (5)$$

where  $f_i = \left(\frac{1-\alpha_i}{x_i^* + \gamma_i}\right)$ . Note that this probability can collapse to the standard conditional logit model, when only one alternative is chosen ( $M = 1$ ), and there are no satiation effects ( $\alpha_k = 1$  for all  $k$ ), as shown in Bhat's study (2008).

### 3.3 The integer programing extreme value (IPEV) model

If the assumption of continuous consumption is not satisfied, then the consumer's problem cannot be solved using the KKT condition. For example, this assumption is violated by the recreation demand. Recreational trips can only be numbered as non-negative integers; thus, the consumer's problem should be written as a non-linear integer programing problem. In this case, consumer's utility maximization is as follows:

$$\text{Max } U(x) \quad \text{s.t.} \quad \sum_{k=1}^K p_k x_k \leq E, x_1 > 0, \text{ and } x_k \in \mathbb{Z}_+ \text{ for all } k \geq 2, \quad (6)$$

where  $\mathbb{Z}_+$  is the set of non-negative integers.

An integer programing problem is generally known to be NP-hard, and it may be difficult to find its exact solution. Approximation algorithms, such as the local search, greedy method, genetic algorithm, simulated annealing, and tabu search, have been proposed as alternatives to the above approach. We focus on the local search algorithm, which iteratively attempts to identify the local optima within a given neighborhood (Aarts and Lenstra, 2003). This algorithm proceeds as follows. Let  $x^1$  be the starting value of consumption. The neighborhood of  $x^1$  with radius  $\delta$  is defined as  $N^1(x^1, \delta) = \{s \in \mathbb{Z}_+ | d(x^1, s) \leq \delta\}$ , where  $d(x^1, s)$  is the Euclidean distance between  $x^1$  and  $s$ . If there exists another consumption  $x^2 \in N^1(x^1)$  that yields  $U(x^2) > U(x^1)$ , then consider the updated neighborhood  $N^2(x^2, \delta) = \{s \in \mathbb{Z}_+ | d(x^2, s) \leq \delta\}$ , and iterate this process. If there is no allocation that satisfies  $x^{n+1} \in N^n(x^n, \delta)$  and  $U(x^{n+1}) > U(x^n)$ , then define the consumption  $x^n$  as the approximate solution of the integer programing problem.

Assume that the first good  $x_1$  is the Hicksian composite good with  $p_1 = 1$  and  $x_1 > p_k$  for all  $k$ . Let  $\tilde{x} = (\tilde{x}_1, \dots, \tilde{x}_K)$  be the solution of (6) using the local search algorithm with radius 1. Then, the approximate local optimal conditions are as follows:

$$\begin{aligned} U(\tilde{x}_k^+) &\leq U(\tilde{x}) \text{ and } U(\tilde{x}_k^-) \leq U(\tilde{x}) \text{ if } \tilde{x}_k > 0, \\ U(\tilde{x}_k^+) &\leq U(\tilde{x}) \text{ if } \tilde{x}_k = 0, \end{aligned} \quad (7)$$



where  $\tilde{x}_k^+ = (\tilde{x}_1 - p_k, \tilde{x}_2, \dots, \tilde{x}_{k-1}, \tilde{x}_k + 1, \tilde{x}_{k+1}, \dots, \tilde{x}_K)$  and  $\tilde{x}_k^- = (\tilde{x}_1 + p_k, \tilde{x}_2, \dots, \tilde{x}_{k-1}, \tilde{x}_k - 1, \tilde{x}_{k+1}, \dots, \tilde{x}_K)$ . As shown in Appendix A, for the additively separable utility function of (1), this condition can be rewritten as

$$\begin{aligned} V_{1,k}^- - V_k^- + \varepsilon_1 &\leq \varepsilon_k \leq V_{1,k}^+ - V_k^+ + \varepsilon_1 \text{ if } \tilde{x}_k > 0, \\ V_k^+ + \varepsilon_k &\leq V_{1,k}^+ + \varepsilon_1 \text{ if } \tilde{x}_k = 0, \end{aligned} \quad (8)$$

where

$$\begin{aligned} V_{1,k}^+ + \varepsilon_1 &= \ln[U_1(\tilde{x}_1) - U_1(\tilde{x}_1 - p_k)] = -\ln \alpha_1 + \ln[\tilde{x}_1^{\alpha_1} - (\tilde{x}_1 - p_k)^{\alpha_1}] + \varepsilon_1, \\ V_{1,k}^- + \varepsilon_1 &= \ln[U_1(\tilde{x}_1 + p_k) - U_1(\tilde{x}_1)] = -\ln \alpha_1 + \ln[(\tilde{x}_1 + p_k)^{\alpha_1} - \tilde{x}_1^{\alpha_1}] + \varepsilon_1 \\ V_k^+ + \varepsilon_k &= \ln[U_k(\tilde{x}_k + 1) - U_k(\tilde{x}_k)] \\ &= \beta' z_k + \ln \gamma_k - \ln \alpha_k + \ln \left[ \left( \frac{\tilde{x}_k + 1}{\gamma_k} + 1 \right)^{\alpha_k} - \left( \frac{\tilde{x}_k}{\gamma_k} + 1 \right)^{\alpha_k} \right] + \varepsilon_k, \text{ and} \\ V_k^- + \varepsilon_k &= \ln[U_k(\tilde{x}_k) - U_k(\tilde{x}_k - 1)] \\ &= \beta' z_k + \ln \gamma_k - \ln \alpha_k + \ln \left[ \left( \frac{\tilde{x}_k}{\gamma_k} + 1 \right)^{\alpha_k} - \left( \frac{\tilde{x}_k - 1}{\gamma_k} + 1 \right)^{\alpha_k} \right] + \varepsilon_k. \end{aligned}$$

Specifically,  $V_k^+$  and  $V_k^-$  are the deterministic components in the log transformations of the utility difference obtained by increasing or decreasing one unit in  $k$  good, respectively.  $V_{1,k}^+$  and  $V_{1,k}^-$  are those of the first outside good obtained by increasing or decreasing one unit of  $k$  good, respectively. Note that the local optimal condition (8) of the integer programming model is similar to the KKT condition (4) of the MDCEV model.

Consider an individual who consumes only the first  $M$  of  $K$  goods. Assume that  $\varepsilon_k$  is the type I extreme value distribution with the inverse scale parameter  $\sigma$  for all  $k$ . The probability of observing  $\tilde{x}^* = (\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_M^*, 0, \dots, 0)$  can be calculated as

$$\begin{aligned} &P(\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_M^*, 0, \dots, 0) \\ &= \int_{\varepsilon_1 = -\infty}^{\varepsilon_1 = +\infty} \left\{ \prod_{k=2}^M [\Pr(V_{1,k}^- - V_k^- + \varepsilon_1 \leq \varepsilon_k \leq V_{1,k}^+ - V_k^+ + \varepsilon_1)] \prod_{k=M+1}^K [\Pr(\varepsilon_k \leq V_{1,k}^+ - V_k^+ + \varepsilon_1)] \right\} \frac{1}{\sigma} g\left(\frac{\varepsilon_1}{\sigma}\right) d\varepsilon_1 \end{aligned} \quad (9)$$

$$\begin{aligned}
&= \int_{\varepsilon_1 = -\infty}^{\varepsilon_1 = +\infty} \left\{ \prod_{k=2}^M \left[ G \left( \frac{V_{1,k}^+ - V_k^+ + \varepsilon_1}{\sigma} \right) \right. \right. \\
&\quad \left. \left. - G \left( \frac{V_{1,k}^- - V_k^- + \varepsilon_1}{\sigma} \right) \right] \prod_{k=M+1}^K G \left( \frac{V_{1,k}^+ - V_k^+ + \varepsilon_1}{\sigma} \right) \right\} \frac{1}{\sigma} g \left( \frac{\varepsilon_1}{\sigma} \right) d\varepsilon_1,
\end{aligned}$$

where  $G(t) = \exp(-\exp(-t))$  and  $g(t) = \exp(-t) \cdot \exp(-\exp(-t))$  are the distribution and density function of the type I extreme value, respectively.

As Appendix B shows, the probability in Equation (9) has a closed-form expression:

$$P(\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_M^*, 0, \dots, 0) = \sum_{s_1=1}^1 \sum_{s_2=0}^1 \dots \sum_{s_M=0}^1 \frac{\prod_{k=1}^M (-1)^{1-s_k}}{\sum_{k=1}^K e^{-\frac{d_k^*}{\sigma}}}, \quad (10)$$

where  $d_k^*$  is the adjusted utility difference between good 1 and  $k$  such that  $d_1^* = 0$  and  $d_k^* = 1[k \leq M] \cdot [s_k \cdot (V_{1,k}^+ - V_k^+) + (1 - s_k) \cdot (V_{1,k}^- - V_k^-)] + 1[k > M] \cdot (V_{1,k}^+ - V_k^+)$  for all  $k \geq 2$ , where  $1[k \leq M]$  is an indicator function equal to one if  $k \leq M$  and zero otherwise. This probability can be rewritten as

$$P(\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_M^*, 0, \dots, 0) = \sum_{s_1=1}^1 \sum_{s_2=0}^1 \dots \sum_{s_M=0}^1 \left[ \prod_{k=1}^M (-1)^{1-s_k} \frac{e^{\frac{v_1}{\sigma}}}{\sum_{k=1}^K e^{\frac{v_k^*}{\sigma}}} \right], \quad (11)$$

where  $v_1 = \beta' z_1 + \ln \gamma_1 - \ln \alpha_1 - \ln p_1$  and  $v_k^* = v_1 - d_k^*$ .

This closed-form probability has  $2^{M-1}$  terms. Therefore, for a large choice set, calculating probability using this equation can be difficult. Alternatively, the parameters can be estimated using a simulated probability (Train, 2009). Let  $\varepsilon_1^r$  be the  $r$ th draw from the type I extreme value with the inverse scale  $\sigma$ . The simulated probability of observing  $\tilde{x}$  is

$$\begin{aligned}
&\check{P}(\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_M^*, 0, \dots, 0) \\
&= \frac{1}{R} \sum_{r=1}^R \left\{ \prod_{k=2}^M \left[ G \left( \frac{V_{1,k}^+ - V_k^+ + \varepsilon_1^r}{\sigma} \right) \right. \right. \\
&\quad \left. \left. - G \left( \frac{V_{1,k}^- - V_k^- + \varepsilon_1^r}{\sigma} \right) \right] \prod_{k=M+1}^K G \left( \frac{V_{1,k}^+ - V_k^+ + \varepsilon_1^r}{\sigma} \right) \right\}, \quad (12)
\end{aligned}$$

where  $R$  is the number of draws.

### 3.4 Mixed IPEV model

The IPEV model assumes that the error component of  $\varepsilon$  is the iid extreme value distribution. The mixed IPEV model, which introduces a mixed distribution to the IPEV model, can incorporate more general error structures. Assume that the log transformations of the utility differences have the additional error components  $\eta$ :  $V_k^+(\eta) + \varepsilon_k$ ,  $V_k^-(\eta) + \varepsilon_k$ ,  $V_{1,k}^+(\eta) + \varepsilon_1$ , and  $V_{1,k}^-(\eta) + \varepsilon_1$ . Let  $f(\eta)$  be the density of  $\eta$ . The probability of observing  $\tilde{x}^*$  in the mixed IPEV model is as follows:

$$P(\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_M^*, 0, \dots, 0) = \int_{\eta} \sum_{s_1=1}^1 \sum_{s_2=0}^1 \dots \sum_{s_M=0}^1 \frac{\prod_{k=1}^M (-1)^{1-s_k}}{\sum_{k=1}^K e^{-\frac{d_k^*(\eta)}{\sigma}}} f(\eta) d\eta, \text{ where} \quad (13)$$

$$d_k^*(\eta) = 1[k \leq M] \left[ s_k \left( V_{1,k}^+(\eta) - V_k^+(\eta) \right) + (1 - s_k) \left( V_{1,k}^-(\eta) - V_k^-(\eta) \right) \right]$$

$$+ 1[k > M] \left( V_{1,k}^+(\eta) - V_k^+(\eta) \right).$$

This probability does not have closed-form. However, it can be approximated through simulations. Let  $\eta^r$  be the  $r$ th draw from  $f(\eta)$ . The simulated probability is as follows:

$$\tilde{P}(\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_M^*, 0, \dots, 0) = \frac{1}{R} \sum_{r=1}^R \sum_{s_1=1}^1 \sum_{s_2=0}^1 \dots \sum_{s_M=0}^1 \frac{\prod_{k=1}^M (-1)^{1-s_k}}{\sum_{k=1}^K e^{-\frac{d_k^*(\eta^r)}{\sigma}}}, \quad (14)$$

where  $R$  is the number of draws.

### 3.5 Comparison with discrete choice model

Consider the special case of  $M = 1$  (i.e., only one alternative is chosen). Namely,  $\tilde{x}_1^* > 0$  and  $\tilde{x}_k^* = 0$  for all  $k \geq 2$ . In this case, the probability of Equation (11) collapses to the standard logit for the discrete choice model:

$$P(\tilde{x}_1^*, 0, \dots, 0) = \frac{e^{\frac{v_1}{\sigma}}}{\sum_{k=1}^K e^{\frac{v_k^*}{\sigma}}}, \quad (15)$$

where

$$\begin{aligned}
v_1 &= \beta' z_1 + \ln \gamma_1 - \ln \alpha_1 - \ln p_1, \\
v_k^* &= \beta' z_k + \ln \gamma_k - \ln \alpha_k + \ln \left[ \left( \frac{\tilde{x}_k + 1}{\gamma_k} + 1 \right)^{\alpha_k} - \left( \frac{\tilde{x}_k}{\gamma_k} + 1 \right)^{\alpha_k} \right] - \ln [\tilde{x}_1^{\alpha_1} - (\tilde{x}_1 - p_k)^{\alpha_1}].
\end{aligned} \tag{16}$$

When  $\alpha_k = \gamma_k = 1$ , this probability can be reduced to a conditional logit model:

$$P(\tilde{x}_1^*, 0, \dots, 0) = \frac{e^{b' z_1 - c \cdot \ln p_1}}{\sum_{k=1}^K e^{b' z_k - c \cdot \ln p_k}}, \tag{17}$$

where  $b = \beta/\sigma$  and  $c = 1/\sigma$ .

#### 4. Demand prediction and welfare analysis with integer demand

##### 4.1 The continuous demand model

Demand prediction requires solving the utility-maximization problem. Let  $x^*(p, z, E, \varepsilon)$  be the demand function that is the solution to the problem and  $V(p, z, E, \varepsilon) = U(x^*(p, z, E, \varepsilon))$  be the indirect utility function. For continuous demand, Pinjari and Bhat (2021) show that there exists an analytical solution to  $x^*(p, z, E, \varepsilon)$ , when all the satiation parameters of the utility function (1) are equal ( $\alpha_k = \alpha$  for all  $k$ ), and if the chosen and non-chosen alternatives are known. However, in general, no closed-form solution to  $x^*(p, z, E, \varepsilon)$  exists; therefore, the demand prediction requires numerical methods. Pinjari and Bhat (2021) propose demand forecasting method using the bisection algorithm over  $\lambda$  (the Lagrange multiplier) with the KKT condition.

For welfare analysis, the compensating variation (CV) associated with the change from  $(p^0, z^0)$  to  $(p^1, z^1)$  is the solution to

$$V(p^0, z^0, E, \varepsilon) = V(p^1, z^1, E - CV, \varepsilon). \tag{18}$$

von Haefen et al. (2004) develop a nesting bisection algorithm to solve the CV using the KKT condition. von Haefen (2007) proposes a bisection algorithm to solve the expenditure minimization problem. Lloyd-Smit (2018) demonstrated a more efficient method for solving the CV.

##### 4.2 The integer demand model

For integer demand, Pinjari and Bhat's (2021) strategy for demand prediction cannot be applied because their algorithm uses the KKT first-order condition, which is not satisfied under the constraint of integer demand. Alternatively, we propose using the greedy method to find a solution to the utility-maximization problem. The greedy method was developed using integer programming literature (Aarts and

Lenstra, 2003). This algorithm makes a locally optimal choice at each iteration. The greedy method is an approximation algorithm that does not always yield the optimal solutions. However, this approach performs well in many maximization problems in which the target function is globally concave.

Let  $\tilde{x}(p, z, E, \varepsilon)$  be the integer demand function that is the solution to the integer programming problem. The greedy strategy for finding the solution  $\tilde{x}$  conditional on the error components  $\varepsilon$  is as follows.

Algorithm 1: the greedy method of demand solution

1. Let  $\tilde{x}^t$  be the demand at iteration  $t$ . Set the initial values  $\tilde{x}^1 = (E, 0, \dots, 0)$ .
2. Construct the candidate  $c_k^t = (\tilde{x}_1^t - p_k, \tilde{x}_2^t, \dots, \tilde{x}_{k-1}^t, \tilde{x}_k^t + 1, \tilde{x}_{k+1}^t, \dots, \tilde{x}_K^t)$  for all  $k \geq 2$  and  $c_1^t = \tilde{x}^t$ .
3. If there exists locally optimal consumption  $c_k^t$ , such that  $U(c_k^t | \varepsilon) > U(c_j^t | \varepsilon)$  for all  $j$ , set  $\tilde{x}^{t+1} = c_k^t$ , and repeat step 2 and 3. Otherwise, stop the iteration and keep  $\tilde{x}^t$ .

This procedure will yield the approximate optimal consumption  $\tilde{x}$  conditional on  $\varepsilon$  under the constraints of non-negative and integer consumption.  $x_1$  is a continuous variable; therefore, a combination of the greedy method and numerical bisection algorithm can be used to solve the CV associated with the change from  $(p^0, z^0)$  to  $(p^1, z^1)$ , which is defined by  $\tilde{V}(p^0, z^0, E, \varepsilon) = \tilde{V}(p^1, z^1, E - CV, \varepsilon)$ . The search algorithm for a CV with an integer demand is as follows.

Algorithm 2: the bisection method for searching CV

1. Calculate the baseline utility,  $\tilde{V}^0 = \tilde{V}(p^0, z^0, E, \varepsilon)$ .
2. At iteration  $t$ , set  $CV_a^t = (CV_l^{t-1} + CV_u^{t-1})/2$ . To initialize the algorithm, set  $CV_l^0 = -E$  and  $CV_u^0 = E$ .
3. Conditional on  $(p^1, z^1, E - CV_a^t, \varepsilon)$ , solve for  $\tilde{x}^t$  using the greedy method of algorithm 1.
4. Construct the utility  $\tilde{V}^t = \tilde{V}(p^1, z^1, E - CV_a^t, \varepsilon)$ . If  $\tilde{V}^t > \tilde{V}^0$ , set  $CV_l^t = CV_a^t$  and  $CV_u^t = CV_u^{t-1}$ . Otherwise, set  $CV_l^t = CV_l^{t-1}$  and  $CV_u^t = CV_a^t$ .
5. Iterate until convergence.

The predicted demand  $\tilde{x}(\varepsilon)$  and  $CV(\varepsilon)$  have the error components  $\varepsilon$ . The analyst can estimate the expectation of demand  $E[\tilde{x}]$  and the compensating variation  $E[CV]$  using a simulation. First,  $\varepsilon_1$  of the first good is drawn from the type I extreme value distribution with the inverse scale  $\sigma$ . Let  $\varepsilon_1^t$  be the  $t$ th draw. Second,  $(\varepsilon_2, \dots, \varepsilon_K)$  of other goods are drawn using the procedure proposed by von Haefen et al. (2004). From the approximated local optimal condition of (8), if the observed demand is the corner solution ( $\tilde{x}_k^* = 0$ ), the error component  $\varepsilon_k$  has the truncated distribution of  $\varepsilon_k^t < V_{1,k}^+ - V_k^+ + \varepsilon_1^t$ . Otherwise ( $\tilde{x}_k^* > 0$ ), it has the truncated distribution of  $V_{1,k}^- - V_k^- + \varepsilon_1^t \leq \varepsilon_k^t \leq V_{1,k}^+ - V_k^+ + \varepsilon_1^t$ . Therefore,  $\varepsilon_k^t$  for all  $k \geq 2$  can be simulated by the following truncated type I extreme value distribution:

$$\begin{aligned}
\varepsilon_k^t &= -\ln \left\{ -\ln \left[ G \left( \frac{V_{1,k}^+ - V_k^+ + \varepsilon_1^t}{\sigma} \right) \mu^t \right] \right\} \sigma, \text{ for } \tilde{x}_k^* = 0, \\
\varepsilon_k^t &= -\ln \left\{ -\ln \left[ (1 - \mu^t) G \left( \frac{V_{1,k}^+ - V_k^+ + \varepsilon_1^t}{\sigma} \right) + \mu^t G \left( \frac{V_{1,k}^- - V_k^- + \varepsilon_1^t}{\sigma} \right) \right] \right\} \sigma, \text{ for } \tilde{x}_k^* > 0,
\end{aligned} \tag{19}$$

where  $\mu^t$  is the  $t$ th draw from the standard uniform distribution. The simulated demand  $\tilde{x}(\varepsilon^t)$  and CV  $CV(\varepsilon^t)$  can be calculated using the generated draws  $\varepsilon^t$ . Iterating this process and averaging the results provide  $E[\tilde{x}]$  and  $E[CV]$ .

For the mixed IPEV model, demand prediction and welfare analysis require the error components of  $\varepsilon$  and  $\eta$ . The joint distribution of the error components  $f(\varepsilon, \eta | \tilde{x}^*)$  given observed demand  $\tilde{x}^*$  can be decomposed as  $f(\varepsilon, \eta | \tilde{x}^*) = f_1(\eta | \tilde{x}^*) \cdot f_2(\varepsilon | \eta, \tilde{x}^*)$ . von Haefen et al. (2004) proposes an adaptive Metropolis-Hastings algorithm to simulate from  $f(\varepsilon, \eta | \tilde{x}^*)$ . After drawing  $\eta$ , the random draws of  $\varepsilon$  can be simulated using Equation (19).

## 5. Empirical illustration

### 5.1 Example of small choice set

To illustrate the proposed IPEV model, we use data on recreational trips to the national parks in Japan. The first empirical study analyzes the trip data to the national parks in Hokkaido, a northern island of Japan. Hokkaido has six national parks, including Shiretoko, a World Heritage Site. To investigate the number of trips to each national park in Hokkaido, we conducted an online survey of Hokkaido residents. The respondents were asked about the number of trips they made to national parks from July to November 2008. The sample consisted of 763 respondents with information on 392 trips. Table 1 presents the trip distributions of the data. Note that while 69% respondents did not visit any national parks in Hokkaido, 31% made one or more trips.

We used six national parks as the choice set. Table 2 provides the descriptive statistics of the data. *Price* is the two-way trip cost between the respondent's residence and each park, which includes expressway toll, vehicle operation cost (US\$0.155 per mile), and opportunity cost.<sup>3</sup> *Vehicle regulation* is defined as the distance of roads where private vehicle access is prohibited to protect the natural landscape.

Table 1 Trip distribution of the small choice set

Table 2 Descriptive statistics of the small choice set

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<sup>3</sup> The vehicle operation cost is calculated using an average of gasoline prices, gas mileage, and exchange rate at 2008 (US\$ 1 = 104 yen). We calculate the opportunity cost using the method proposed by Cesario (1976): The opportunity cost = two-way trip time \* wage rate / 3.

As shown in Bhat (2008), identifying both  $\alpha_k$  and  $\gamma_k$  parameters is difficult. In our study, following Kuriyama et al. (2020), we use  $\alpha\gamma$ -profile utility function:

$$U(x) = \frac{\psi_1}{\alpha_1} x_1^{\alpha_1} + \sum_{k=2}^K \gamma \psi_k \ln \left( \frac{x_k}{\gamma} + 1 \right), \quad (20)$$

$$\psi_1 = \exp(\varepsilon_1) \text{ and } \psi_k = \exp(\beta' z_k + \varepsilon_k).$$

Note that this function form is a combination of the  $\alpha$ -profile for the Hicks composite good  $x_1$  and the  $\gamma$ -profile for other goods. The baseline utility component  $\psi_k$  includes site attributes as well as individual attributes. Satiation parameters  $\gamma_k$  are assumed to be same for all goods because it is difficult to identify both the parameter of site attributes and satiation parameters for each site.

Using these data, we compare the MDCEV model assuming continuous demand with the IPEV model assuming integer demand. For the IPEV model, we consider an estimation using a closed-form and simulated probabilities. Six choice sets were used for the data. Therefore, the maximum number of terms in the closed-form probability (10) is  $2^6 = 64$ , and it is feasible to calculate the closed-form probability for this small choice set data.

Table 3 provides the estimation results using the three models: (1) the MDCEV, (2) the IPEV using the closed-form probability, and (3) the IPEV using the simulated probability. The parameters of the MDCEV and IPEV1 (closed-form) models are estimated using the maximum log likelihood, whereas the parameters of the IPEV2 (simulation) model are estimated using the maximum simulated log likelihood with 200 Halton draws, following Sikder and Pinjari (2013). This table shows the large difference in the parameter values between the MDCEV and IPEV models. However, the estimated results of the IPEV1 model are remarkably close to those of the IPEV2 model. This suggests that the simulated maximum likelihood of the IPEV model performs well in the approximation of the standard maximum likelihood of the IPEV model using the closed-form probability. Furthermore, the log-likelihood of this table suggests that the IPEV models are better fit to the data than the MDCEV model.

Table 3 Estimation result of the small choice set

For demand prediction and welfare analysis, we consider two scenarios: (a) closing Shiretoko National Park, a World Heritage Site, and (b) doubling vehicle regulation. In 2005, the Shiretoko area was designated a UNESCO World Heritage Site. Regarding scenario (a), Shiretoko Five Lakes, the most popular area for visitors, is closed in some seasons because it is a natural habitat of the brown bear, and the park manager needs to prevent accidents due to encounters between visitors and bears. In scenario (b),

road congestion in the parks had a negative impact on their natural environment. Although the Ministry of the Environment has regulated private vehicle access in some parks, a decrease in visitors may have negative effects on the local economy.

Tables 4 and 5 provide the demand prediction and compensation variations of the two scenarios using the MDCEV and IPEV models with small choice set data. Table 4 shows that there are no significant differences in the demand prediction using the three models. One possible reason for this result is averaging by rounding up and down the integer demand. For example, consider two respondents who consume 1.6 and 2.4 trips in continuous demand. The average number of trips of continuous demand over two people was 2.0. In integer demand, 1.6 trip rounds up and 2.4 trip rounds down. Thus, the average number of trips for integer demand is also 2.0.

Table 4 Demand prediction of the small choice set

Table 5 Compensating variations of the small choice set

In contrast, Table 5 suggests that there are significant differences in the CVs between the MDCEV and IPEV models. The estimated CVs show that the welfare loss of the integer demand models (IPEV1 and IPEV2) is larger than that of the continuous demand model (MDCEV). The utility maximization problem of the integer demand model has an additional constraint on non-negative integer consumption. This constraint may cause greater welfare losses in the IPEV model. For example, consider an individual who visits the closest site only once. If the visited site is close, the individual can visit a distant site. However, the number of trips to distant sites may decrease because of higher costs. For continuous demand, it may be less than one, but a positive number, say 0.4 trips. In contrast, it may be zero for integer demand and the welfare loss of site closure with integer demand may be larger than that with continuous demand. Thus, for integer demand, the MDCEV model that assumes continuous demand may underestimate the welfare loss owing to site closure.

## 5.2 Example of large choice set

The second empirical study uses trip data to all national parks in Japan. Japan has 34 national parks. In 2019, the national parks had more than 369 million visitors. We conducted a nationwide web survey in February 2020 to analyze the recreation demand for national parks. The respondents were asked about their number of visits to each national park in 2019. The sample consisted of 1591 respondents with information on 3,285 trips. Table 6 presents the trip distribution for our data. Note that while 62% respondents did not visit any national park in 2019, 38% visited national parks one or more times.

Table 6 Trip distribution of the large choice set

Table 7 Descriptive statistics of the large choice set



We consider the 34 national parks as the choice set.<sup>4</sup> Table 7 demonstrates the descriptive statistics for the data. *Price* is the two-way trip cost between the respondent's residence and each park, which includes airfare, railroad fare, expressway toll, vehicle operation cost (US\$0.219 per mile), and opportunity cost.<sup>5</sup> *World Heritage* is a dummy variable for the World Heritage Sites. Vehicle access is regulated in some parks due to traffic and damage to the natural environment. *Vehicle regulation* is a dummy variable that prohibits private vehicle access to parks. *Area 1 – Area 7* represent the area dummies. The Japanese Ministry of the Environment classifies national parks into seven categories.

In this empirical analysis, as in the study on small choice set, we use  $\alpha\gamma$ -profile utility function. Using these data, we compare the MDCEV model assuming continuous demand with the IPEV model assuming integer demand. For the IPEV model, it may not be feasible to estimate using a closed-form probability because of the large choice set. Thirty-four choice sets were used for the data. The maximum number of the terms in the closed-form probability (10) is  $2^{34} \approx 1.7 \times 10^{10}$ . Therefore, we use the IPEV model with a simulated probability.

Table 8 presents the estimated parameters of the two models. The parameters of the MDCEV model are estimated using the maximum log-likelihood, whereas those of the IPEV (simulation) model are estimated using the maximum simulated log-likelihood with 200 Halton draws. This table shows the large difference in the parameter values between the MDCEV and IPEV models. The log-likelihood of this table suggests that the IPEV model is a better fit for the data than the MDCEV model.

Table 8 Estimation result of the large choice set

We consider two scenarios for demand prediction and welfare analysis: (a) closing visitor centers and (b) closing Fuji-Hakone-Izu National Park. In scenario (a), on April 7, 2020, the Japanese government declared a state of emergency owing to COVID-19. Following the Japanese government's declaration of emergency, most visitor centers in national parks were closed to avoid infection. In scenario (b), Fuji-Hakone-Izu National Park is the most popular national park in Japan. Mount Fuji was included on the UNESCO World Heritage List in 2013. Mount Fuji has not erupted since 1707, and at present, the Japan Meteorological Agency has assigned Mount Fuji the lowest threat level for volcanic eruptions. However, the closure of the national park owing to the eruption of Mount Fuji might have a large and negative

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<sup>4</sup> Kuriyama et al. (2020) used 32 national parks as the choice set. In their analysis, two national parks, Yambaru and Amami-Gunto, were excluded from the choice set because they were established after their survey was conducted.

<sup>5</sup> The vehicle operation cost is calculated using an average of gasoline prices, gas mileage, and exchange rate in 2019 (US\$ 1 = 110 yen).

impact on climbers, even if the risk of eruption is extremely low. Mount Hakone is also located in the park; in May 2015, Owakudani, the valley beneath Mount Hakone, was closed because of volcanic activity.

The estimated demand prediction and CVs in the two scenarios with large choice set data using the MDCEV and IPEV models are demonstrated in Tables 9 and 10, respectively. The results of the large choice set data are similar to those of the small choice set data. While Table 9 shows no significant differences in demand prediction between the MDCEV and IPEV models, the welfare loss estimated using the IPEV model is significantly larger than that estimated using the MDCEV model.

Table 9 Demand prediction of the small choice set

Table 10 Compensating variations of the small choice set

## 6. Discussion

In this study, we propose an IPEV model that applies the integer programming method to the MDCEV model and accounts for non-negative integer demand with multiple alternatives. Our proposed model is appealing as it satisfies the constraint imposed by the non-negative integer amount of consumption, in addition to having the same advantages as the MDCEV continuous demand approach. The IPEV model provides a single structural framework that simultaneously models the consumer choice situation and purchase quantity decision with the constraint of non-negative integer consumption, while allowing for the possibility of a corner solution, in addition to being consistent with the utility theory. Furthermore, the IPEV model has a closed-form expression for the probability of observing consumption. It may not be feasible to use the closed-form probability of the IPEV model for a large choice set. To address the calculation burden, we propose an IPEV model with a simulated probability for large choice set data.

In demand prediction and welfare analysis, finding exact solutions to integer programming problems is generally difficult. To overcome these difficulties, we suggested the use of alternative approximate approaches, including the local search algorithm and greedy method, both of which have been widely used in other integer programming studies. With an empirical application using the data on recreational trips to national parks in Japan with small and large choice sets, we compared our proposed IPEV model with the continuous demand MDCEV model. The empirical results suggested that the integer programming approach generally provides a better fit for our data than the continuous demand approach. The first example, using the small choice set, showed that the estimated results of the IPEV model with closed-form probability were remarkably close to those of the IPEV model with simulation probability. Moreover, the welfare loss estimated using the MDCEV model is significantly lower than that estimated using the IPEV model. This result suggests that ignoring the integer property of demand may cause an underestimation of welfare loss.

We propose two additional investigation paths for the proposed IPEV model. First, the heterogeneity of the preferences in the IPEV model should be investigated. For multiple discrete-continuous demands, several models have been proposed to implement heterogeneity, including the random parameter KT (von

Haefen and Phaneuf, 2005), mixed MDCEV (Bhat, 2008), latent segmentation KT (Kuriyama et al., 2010), and latent segmentation MDCEV (Wafa et al., 2015). The random parameter and latent segmentation approaches for the IPEV model may be worth investigating. Second, an IPEV model with multiple constraints should be examined. The IPEV model considers an individual's decision regarding consumption allocation within budget constraints. However, time may also be a fundamental constraint on leisure trips. Castro et al. (2012) proposed an MDCEV model with double constraints (budget and time) for continuous demand, and Kuriyama et al. (2020) considered an MDCEV model with triple constraints (budget, weekend time, and long holiday time). The development of an econometric model using an IPEV with multiple constraints is an important issue for the future.

## Appendix A. The approximated locally optimal condition

First, consider  $U(\tilde{x}_k^+) \leq U(\tilde{x})$ . For the additively separable utility function defined in Equation (1),  $U(\tilde{x}_k^+) \leq U(\tilde{x})$  can be written as

$$\frac{\psi_1}{\alpha_1} (\tilde{x}_1 - p_k)^{\alpha_1} + \frac{\gamma_k \psi_k}{\alpha_k} \left\{ \left( \frac{\tilde{x}_k + 1}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\} \leq \frac{\psi_1}{\alpha_1} \tilde{x}_1^{\alpha_1} + \frac{\gamma_k \psi_k}{\alpha_k} \left\{ \left( \frac{\tilde{x}_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\}. \quad (\text{A1})$$

It can be rewritten as

$$\frac{\gamma_k \psi_k}{\alpha_k} \left\{ \left( \frac{\tilde{x}_k + 1}{\gamma_k} + 1 \right)^{\alpha_k} - \left( \frac{\tilde{x}_k}{\gamma_k} + 1 \right)^{\alpha_k} \right\} \leq \frac{\psi_1}{\alpha_1} \{ \tilde{x}_1^{\alpha_1} - (\tilde{x}_1 - p_k)^{\alpha_1} \}. \quad (\text{A2})$$

The left-hand side is the utility difference of  $U_k(\tilde{x}_k + 1) - U_k(\tilde{x}_k)$ , and the right-hand side is the difference in  $U_1(\tilde{x}_1) - U_1(\tilde{x}_1 - p_k)$ . Note that  $\psi_1 = \exp(\varepsilon_1)$  and  $\psi_k = \exp(\beta' z_k + \varepsilon_k)$ . Taking the logarithms on both sides, we obtain the following inequality:

$$V_k^+ + \varepsilon_k \leq V_{1,k}^+ + \varepsilon_1, \quad (\text{A3})$$

where

$$\begin{aligned} V_k^+ + \varepsilon_k &= \beta' z_k + \ln \gamma_k - \ln \alpha_k + \ln \left[ \left( \frac{\tilde{x}_k + 1}{\gamma_k} + 1 \right)^{\alpha_k} - \left( \frac{\tilde{x}_k}{\gamma_k} + 1 \right)^{\alpha_k} \right] + \varepsilon_k \\ &= \ln[U_k(\tilde{x}_k + 1) - U_k(\tilde{x}_k)], \text{ and} \\ V_{1,k}^+ + \varepsilon_1 &= -\ln \alpha_1 + \ln[\tilde{x}_1^{\alpha_1} - (\tilde{x}_1 - p_k)^{\alpha_1}] + \varepsilon_1 = \ln[U_1(\tilde{x}_1) - U_1(\tilde{x}_1 - p_k)]. \end{aligned}$$

Then, consider  $U(\tilde{x}_k^-) \leq U(\tilde{x})$ . This inequality can be written as

$$\frac{\psi_1}{\alpha_1} (\tilde{x}_1 + p_k)^{\alpha_1} + \frac{\gamma_k \psi_k}{\alpha_k} \left\{ \left( \frac{\tilde{x}_k - 1}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\} \leq \frac{\psi_1}{\alpha_1} \tilde{x}_1^{\alpha_1} + \frac{\gamma_k \psi_k}{\alpha_k} \left\{ \left( \frac{\tilde{x}_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\}. \quad (\text{A4})$$

It can be expressed as

$$\frac{\psi_1}{\alpha_1} \{ (\tilde{x}_1 + p_k)^{\alpha_1} - \tilde{x}_1^{\alpha_1} \} \leq \frac{\gamma_k \psi_k}{\alpha_k} \left\{ \left( \frac{\tilde{x}_k}{\gamma_k} + 1 \right)^{\alpha_k} - \left( \frac{\tilde{x}_k - 1}{\gamma_k} + 1 \right)^{\alpha_k} \right\}. \quad (\text{A5})$$

Take the logarithm on both sides, it can be rewritten as

$$V_{1,k}^- + \varepsilon_1 \leq V_k^- + \varepsilon_k, \quad (\text{A6})$$

where

$$\begin{aligned} V_{1,k}^- + \varepsilon_1 &= -\ln \alpha_1 + \ln[(\tilde{x}_1 + p_k)^{\alpha_1} - \tilde{x}_1^{\alpha_1}] + \varepsilon_1 = \ln[U_1(\tilde{x}_1 + p_k) - U_1(\tilde{x}_1)], \text{ and} \\ V_k^- + \varepsilon_k &= \beta' z_k + \ln \gamma_k - \ln \alpha_k + \ln \left[ \left( \frac{\tilde{x}_k}{\gamma_k} + 1 \right)^{\alpha_k} - \left( \frac{\tilde{x}_k - 1}{\gamma_k} + 1 \right)^{\alpha_k} \right] + \varepsilon_k \\ &= \ln[U_k(\tilde{x}_k) - U_k(\tilde{x}_k - 1)]. \end{aligned}$$

## Appendix B. Derivation of the closed-form probability for the IPEV model

Let  $d_k^+ = V_{1,k}^+ - V_k^+$  and  $d_k^- = V_{1,k}^- - V_k^-$ . From (9), the probability of observing  $\tilde{x}^* = (\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_M^*, 0, \dots, 0)$  is

$$\begin{aligned} P(\tilde{x}^*) &= \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ \prod_{k=2}^M \left[ G\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right) - G\left(\frac{d_k^- + \varepsilon_1}{\sigma}\right) \right] \prod_{k=M+1}^K G\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right) \right\} \frac{1}{\sigma} g\left(\frac{\varepsilon_1}{\sigma}\right) d\varepsilon_1, \\ &= \sum_{s_2=0}^1 \cdots \sum_{s_M=0}^1 \left[ \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ \prod_{k=2}^M \left[ G\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right) \right]^{s_k} \left[ -G\left(\frac{d_k^- + \varepsilon_1}{\sigma}\right) \right]^{1-s_k} \prod_{k=M+1}^K G\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right) \right\} \frac{1}{\sigma} g\left(\frac{\varepsilon_1}{\sigma}\right) d\varepsilon_1 \right] \\ &= \sum_{s_2=0}^1 \cdots \sum_{s_M=0}^1 \left[ \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ \prod_{k=2}^M \left[ e^{-e^{-\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right)}} \right]^{s_k} \left[ -e^{-e^{-\left(\frac{d_k^- + \varepsilon_1}{\sigma}\right)}} \right]^{1-s_k} \prod_{k=M+1}^K e^{-e^{-\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right)}} \right\} \frac{1}{\sigma} e^{-\frac{\varepsilon_1}{\sigma}} e^{-e^{-\frac{\varepsilon_1}{\sigma}}} d\varepsilon_1 \right] \quad (\text{B1}) \\ &= \sum_{s_1=1}^1 \sum_{s_2=0}^1 \cdots \sum_{s_M=0}^1 \left[ \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ \prod_{k=1}^M \left[ e^{-e^{-\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right)}} \right]^{s_k} \left[ -e^{-e^{-\left(\frac{d_k^- + \varepsilon_1}{\sigma}\right)}} \right]^{1-s_k} \prod_{k=M+1}^K e^{-e^{-\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right)}} \right\} \frac{1}{\sigma} e^{-\frac{\varepsilon_1}{\sigma}} d\varepsilon_1 \right], \end{aligned}$$

where  $d_1^+ = V_{1,1}^+ - V_1^+ = 0$ . This probability can be written as

$$\begin{aligned}
P(\tilde{x}^*) &= \\
&\sum_{s_1=1}^1 \sum_{s_2=0}^1 \cdots \sum_{s_M=0}^1 \left[ \left( \prod_{k=1}^M (-1)^{1-s_k} \right) \right. \\
&\quad \left. \cdot \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ \prod_{k=1}^M e^{-s_k \cdot e^{-\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right)}} e^{-(1-s_k) \cdot e^{-\left(\frac{d_k^- + \varepsilon_1}{\sigma}\right)}} \prod_{k=M+1}^K e^{-e^{-\left(\frac{d_k^+ + \varepsilon_1}{\sigma}\right)}} \right\} \frac{1}{\sigma} e^{-\frac{\varepsilon_1}{\sigma}} d\varepsilon_1 \right] \quad (\text{B2}) \\
&= \sum_{s_1=1}^1 \sum_{s_2=0}^1 \cdots \sum_{s_M=0}^1 \left[ \left( \prod_{k=1}^M (-1)^{1-s_k} \right) \cdot \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ \prod_{k=1}^K e^{-e^{-\left(\frac{d_k^* + \varepsilon_1}{\sigma}\right)}} \right\} \frac{1}{\sigma} e^{-\frac{\varepsilon_1}{\sigma}} d\varepsilon_1 \right] \\
&= \sum_{s_1=1}^1 \sum_{s_2=0}^1 \cdots \sum_{s_M=0}^1 \left[ \left( \prod_{k=1}^M (-1)^{1-s_k} \right) \cdot \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \exp\left(-e^{-\frac{\varepsilon_1}{\sigma}} \sum_{k=1}^K e^{-\left(\frac{d_k^*}{\sigma}\right)}\right) \frac{1}{\sigma} e^{-\frac{\varepsilon_1}{\sigma}} d\varepsilon_1 \right],
\end{aligned}$$

where  $d_k^* = 1[k \leq M] \cdot [s_k \cdot d_k^+ + (1-s_k) \cdot d_k^-] + 1[k > M] \cdot d_k^+$ . Let  $t = e^{-\frac{\varepsilon_1}{\sigma}}$ . Then,  $dt = -\frac{1}{\sigma} e^{-\frac{\varepsilon_1}{\sigma}} d\varepsilon_1$ , and the integral of this expression can be written as

$$\begin{aligned}
&\int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \exp\left(-e^{-\frac{\varepsilon_1}{\sigma}} \sum_{k=1}^K e^{-\left(\frac{d_k^*}{\sigma}\right)}\right) \frac{1}{\sigma} e^{-\frac{\varepsilon_1}{\sigma}} d\varepsilon_1 = \int_0^\infty \exp\left(-t \sum_{k=1}^K e^{-\frac{d_k^*}{\sigma}}\right) dt = \frac{\exp\left(-t \sum_{k=1}^K e^{-\frac{d_k^*}{\sigma}}\right) \Big|_0^\infty}{-\sum_{k=1}^K e^{-\frac{d_k^*}{\sigma}}} \quad (\text{B3}) \\
&= \frac{1}{\sum_{k=1}^K e^{-\frac{d_k^*}{\sigma}}}.
\end{aligned}$$

Thus, we can get the closed-form probability of the IPEV model:

$$P(\tilde{x}^*) = \sum_{s_1=1}^1 \sum_{s_2=0}^1 \cdots \sum_{s_M=0}^1 \frac{\prod_{k=1}^M (-1)^{1-s_k}}{\sum_{k=1}^K e^{-\frac{d_k^*}{\sigma}}}. \quad (\text{B4})$$

### Appendix C. An example of three goods case

The closed-form expression of the probability of the IPEV model for the three goods is as follows:

(a)  $\tilde{x}_1^*, \tilde{x}_2^*$ , and  $\tilde{x}_3^* > 0$

$$\begin{aligned}
 P(\tilde{x}_1^*, \tilde{x}_2^*, \tilde{x}_3^*) &= \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ \left[ G\left(\frac{d_2^+ + \varepsilon_1}{\sigma}\right) - G\left(\frac{d_2^- + \varepsilon_1}{\sigma}\right) \right] \left[ G\left(\frac{d_3^+ + \varepsilon_1}{\sigma}\right) - G\left(\frac{d_3^- + \varepsilon_1}{\sigma}\right) \right] \right\} \frac{1}{\sigma} g\left(\frac{\varepsilon_1}{\sigma}\right) d\varepsilon_1 \\
 &= \frac{1}{e^{-\frac{d_1^+}{\sigma}} + e^{-\frac{d_2^+}{\sigma}} + e^{-\frac{d_3^+}{\sigma}}} - \frac{1}{e^{-\frac{d_1^+}{\sigma}} + e^{-\frac{d_2^+}{\sigma}} + e^{-\frac{d_3^-}{\sigma}}} - \frac{1}{e^{-\frac{d_1^+}{\sigma}} + e^{-\frac{d_2^-}{\sigma}} + e^{-\frac{d_3^+}{\sigma}}} \\
 &\quad + \frac{1}{e^{-\frac{d_1^+}{\sigma}} + e^{-\frac{d_2^-}{\sigma}} + e^{-\frac{d_3^-}{\sigma}}}.
 \end{aligned} \tag{C1}$$

(b)  $\tilde{x}_1^*$  and  $\tilde{x}_2^* > 0, \tilde{x}_3^* = 0$

$$\begin{aligned}
 P(\tilde{x}_1^*, \tilde{x}_2^*, 0) &= \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ \left[ G\left(\frac{d_2^+ + \varepsilon_1}{\sigma}\right) - G\left(\frac{d_2^- + \varepsilon_1}{\sigma}\right) \right] G\left(\frac{d_3^+ + \varepsilon_1}{\sigma}\right) \right\} \frac{1}{\sigma} g\left(\frac{\varepsilon_1}{\sigma}\right) d\varepsilon_1 \\
 &= \frac{1}{e^{-\frac{d_1^+}{\sigma}} + e^{-\frac{d_2^+}{\sigma}} + e^{-\frac{d_3^+}{\sigma}}} - \frac{1}{e^{-\frac{d_1^+}{\sigma}} + e^{-\frac{d_2^-}{\sigma}} + e^{-\frac{d_3^+}{\sigma}}}.
 \end{aligned} \tag{C2}$$

(c)  $\tilde{x}_1^* > 0, \tilde{x}_2^*$  and  $\tilde{x}_3^* = 0$

$$P(\tilde{x}_1^*, 0, 0) = \int_{\varepsilon_1=-\infty}^{\varepsilon_1=+\infty} \left\{ G\left(\frac{d_2^+ + \varepsilon_1}{\sigma}\right) G\left(\frac{d_3^+ + \varepsilon_1}{\sigma}\right) \right\} \frac{1}{\sigma} g\left(\frac{\varepsilon_1}{\sigma}\right) d\varepsilon_1 = \frac{1}{e^{-\frac{d_1^+}{\sigma}} + e^{-\frac{d_2^+}{\sigma}} + e^{-\frac{d_3^+}{\sigma}}}. \tag{C3}$$

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Table 1

Trip distribution of the small choice set

Number of trips made per respondent	The number of observations	
0	522	69%
1	150	20%
2	37	5%
3	25	3%
4	12	2%
5 and over	6	1%
Total	752	100%

Table 2

Descriptive statistics of the small choice set

Variables	Description	Mean	Std. dev.	Min	Max
x	The number of trips per individual	0.521	1.143	0.000	18.000
Price	Two-way travel cost (1,000 yen)	23.666	12.520	0.696	64.724
Male	1/0, 1 = male	0.560	0.497	0.000	1.000
Age	Age of respondent	44.654	12.020	25.000	65.000
Income	Household income (1,000 yen)	5,910	3,231	2,000	20,000
Area	Total area of parks (10,000 ha)	8.440	7.020	2.417	49.290
Vehicle Regulation	The distance of roads where private vehicle access is prohibited (100 km)	6.300	9.231	0.000	85.200

Table 3

Estimation result of the small choice set<sup>a</sup>

Variables	MDCEV		IPEV1 (closed-form)		IPEV2 (simulation) <sup>b</sup>	
	Est.	t-stat.	Est.	t-stat.	Est.	t-stat.
Baseline utility ( $\beta$ )						
Constant	-4.962 ***	-6.55	-2.664 **	-3.00	-2.598 **	-2.81
Male	0.379 **	3.05	0.444 **	3.11	0.453 **	3.07
Age	0.007	1.45	0.009 *	1.79	0.009	1.58
Area	0.049 **	3.10	0.074 ***	4.03	0.074 ***	3.47
Vehicle regulation	-0.022 *	-1.95	-0.039 **	-3.04	-0.039 **	-2.70
Satiation parameters						
$\alpha_1$	0.663 ***	5.85	0.623 ***	5.00	0.624 ***	4.73
$\gamma$	1.146 ***	10.91	0.009	1.40	0.009	1.31
Scale parameter ( $\sigma$ )	0.845 ***	20.62	0.996 ***	15.71	1.007 ***	15.29
Log-likelihood	-1442.58		-1176.53		-1176.59	

<sup>a</sup> \*  $p < .10$ , \*\*  $p < .05$ , \*\*\*  $p < .01$ .<sup>b</sup> Parameters are estimated using simulation with 200 Halton draws.

Table 4

Demand prediction of the small choice set<sup>a</sup>

Scenarios	MDCEV	IPEV1 (closed-form)	Difference 1 <sup>b</sup>	IPEV2 (simulation)	Difference 2 <sup>c</sup>
A: Closing Shiretoko	0.489 (0.003)	0.488 (0.000)	0.001 (0.002)	0.488 (0.000)	0.001 (0.002)
B: Doubling vehicle regulation	0.443 (0.049)	0.439 (0.020)	0.004 (0.030)	0.439 (0.028)	0.004 (0.021)

<sup>a</sup> Average number of trips per person. The values in parentheses are standard errors calculated using the procedure reported by Krinsky and Robb (1986) based on 200 iterations.

<sup>b</sup> Difference 1 = MDCEV – IPEV1.

<sup>c</sup> Difference 2 = MDCEV – IPEV2.

Table 5

Compensating variations of the small choice set<sup>a</sup>

Scenarios	MDCEV	IPEV1 (closed-form)	Difference 1 <sup>b</sup>	IPEV2 (simulation)	Difference 2 <sup>c</sup>
A: Closing Shiretoko	-5.38 (0.38)	-21.68 (2.79)	16.29 (2.69)	-21.52 (2.77)	16.14 (2.70)
B: Doubling vehicle regulation	-11.59 (6.71)	-38.54 (12.27)	26.94 (6.95)	-38.46 (14.78)	26.87 (8.64)

<sup>a</sup> USD per person. The values in parentheses are standard errors calculated using the procedure reported by Krinsky and Robb (1986) and based on 200 iterations.

<sup>b</sup> Difference 1 = MDCEV – IPEV1.

<sup>c</sup> Difference 2 = MDCEV – IPEV2.

Table 6

Trip distribution of the large choice set

Number of trips made per respondent	The number of observations	
0	983	62%
1	214	13%
2	108	7%
3	63	4%
4	50	3%
5	43	3%
6	16	1%
7	14	1%
8	14	1%
9	6	0%
10 and over	80	5%
<b>Total</b>	<b>1591</b>	<b>100.0%</b>



Table 7

Descriptive statistics of the large choice set

Variables	Description	Mean	Std. dev.	Min	Max
x	The number of trips per individual	2.065	6.027	0.000	72.000
Price	Two-way trip cost to each site (1000 yen)	78.381	41.460	1.780	222.587
Male	1/0, 1 = Male	0.503	0.500	0.000	1.000
Income	Household income (1000 yen)	6,261	3,833	2,000	20,000
World heritage	1/0, 1 = World Heritage Site	0.206	0.404	0.000	1.000
Visitor center	The number of visitor centers	1.118	1.795	0.000	8.000
Hot spring	1/0, 1 = Hot spring	0.353	0.478	0.000	1.000
Vehicle regulation	1/0, 1 = Vehicle regulation	0.559	0.497	0.000	1.000
Area 1	1/0, 1 = The park is located in the Hokkaido area	0.177	0.381	0.000	1.000
Area 2	1/0, 1 = The park is located in the Tohoku area	0.088	0.284	0.000	1.000
Area 3	1/0, 1 = The park is located in the Kanto area	0.177	0.381	0.000	1.000
Area 4	1/0, 1 = The park is located in the Chubu area	0.147	0.354	0.000	1.000
Area 5	1/0, 1 = The park is located in the Kinki area	0.088	0.284	0.000	1.000
Area 6	1/0, 1 = The park is located in the Chugoku-Shikoku area	0.088	0.284	0.000	1.000
Area 7	1/0, 1 = The park is located in the Kyushu area	0.294	0.208	0.000	1.000

Table 8

Estimation result of the large choice set<sup>a</sup>

Variables	MDCEV			IPEV (simulation) <sup>b</sup>		
	Est.		t-stat.	Est.		t-stat.
Baseline utility ( $\beta$ )						
Constant	-1.115	***	-11.85	0.586	***	6.24
Male	0.498	***	22.35	0.806	***	33.06
World Heritage	0.903	***	12.47	0.943	***	10.80
Visitor center	0.094	***	5.18	0.103	***	4.72
Hot spring	0.300	***	5.00	0.396	***	5.54
Vehicle regulation	-0.665	***	-10.50	-0.745	***	-9.66
Area 1	0.466	***	8.70	0.515	***	7.83
Area 2	0.151		1.47	0.209	*	1.68
Area 3	-0.722	***	-12.21	-0.534	***	-6.81
Area 4	-0.095		-1.46	0.051		0.65
Area 5	-0.264	***	-3.37	-0.067		-0.70
Area 6	0.261	**	3.13	0.296	**	2.85
Satiation parameters						
$\alpha_1$	0.993	***	95.24	0.989	***	95.77
$\gamma$	1.225	***	22.46	0.032	***	8.34
Scale parameter ( $\sigma$ )	0.785	***	53.82	0.924	***	44.68
Log-likelihood	-9903.05			-8303.92		

<sup>a</sup> \*  $p < .10$ , \*\*  $p < .05$ , \*\*\*  $p < .01$ .<sup>b</sup> Parameters are estimated using simulation with 200 Halton draws.

Table 9

Demand prediction of the large choice set<sup>a</sup>

Scenarios	MDCEV	IPEV (simulation)	Difference <sup>b</sup>
A: Closing visitor centers	1.812 (0.150)	1.867 (0.029)	-0.054 (0.144)
B: Closing Fuji-Hakone-Izu National Park	1.865 (0.169)	1.837 (0.001)	0.027 (0.168)

<sup>a</sup> Average number of trips per person. The values in parentheses are standard errors calculated using the procedure reported by Krinsky and Robb (1986) and based on 200 iterations.

<sup>b</sup> Difference = MDCEV – IPEV.

Table 10

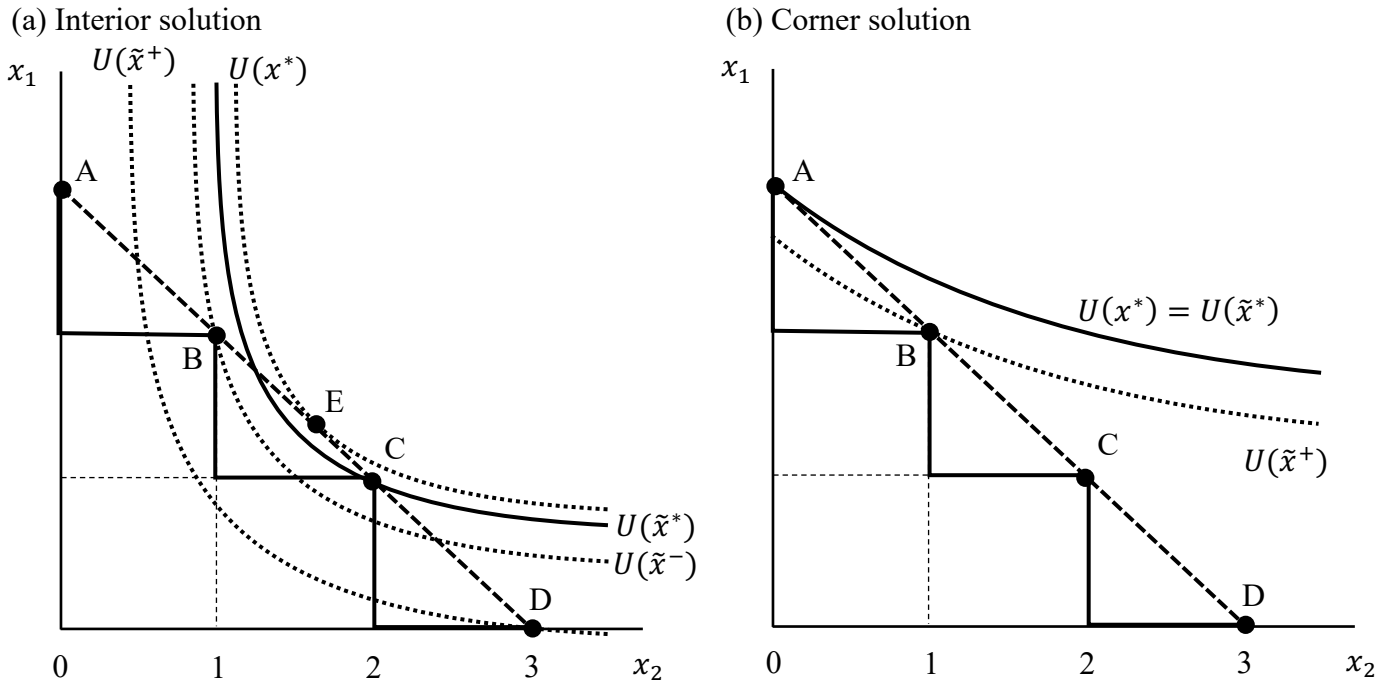
Compensating variations of the large choice set<sup>a</sup>

Scenarios	MDCEV	IPEV (simulation)	Difference <sup>b</sup>
A: Closing visitor centers	-91.37 (12.48)	-209.38 (33.25)	118.00 (24.06)
B: Closing Fuji-Hakone-Izu National Park	-32.76 (10.38)	-103.00 (2.85)	70.24 (11.38)

<sup>a</sup> USD per person. The values in parentheses are standard errors calculated using the procedure reported by Krinsky and Robb (1986) and based on 200 iterations.

<sup>b</sup> Difference = MDCEV – IPEV.

Figure 1. Continuous and integer demand models



Note: This figure shows the optimal consumption of two goods.  $x_1$  is the numeraire good and  $x_2$  is the good of the integer demand. (a) is the case of the interior solution ( $x_2 > 0$ ). (b) is the case of the corner solution ( $x_2 = 0$ ).  $U(x)$  is the indifference curve for the consumption of  $x$ . The optimal consumption under continuous demand is  $x^*$ . The optimal allocation of integer demand is  $\tilde{x}^*$ . Please refer to the text for further details.