The Value of Leisure Time of Weekends and Long Holidays:

The Multiple Discrete–Continuous Extreme Value (MDCEV) Choice Model with Triple Constraints

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Abstract

In this study, we apply the multiple discrete–continuous extreme value (MDCEV) model with triple constraints to identify the value of leisure time during weekends and long holidays. Our approach models the economic behavior of leisure trips with the triple constraints of budget, duration of weekend, and duration of holiday. The econometric model is developed to construct an estimation using the observed allocation of goods and time between a weekend and a long holiday. Our proposed approach models endogenous time allocation and analyzes the substitution effect between weekends and long holidays. Furthermore, we solve a system of nonlinear equations using the Karush-Kuhn-Tucker condition and the Markov Chain Monte Carlo (MCMC) method, which constructs the value of leisure times, demand prediction, and welfare analysis. Finally, we apply the proposed model to the recreation demand for national parks in Japan. The results suggest a significantly large difference in the value of leisure time between weekends and long holidays with low substitution effects between them.

Keywords

Value of time, multiple discrete–continuous models, recreation demand, demand system, welfare analysis

JEL Codes

Q26, Q51, R41

Introduction

Time is one of the fundamental constraints for leisure trips. It is difficult for most workers to take a leisure trip while working, so the possibility of a leisure trip is constrained to non-working time such as a weekend or a long holiday. For a weekend trip, the maximum length of the trip is two days, Saturday and Sunday. When a leisure trip to a faraway destination requires more than two days, a long holiday has to be available. Another basic constraint is money. Consumers have to consider the expenditure allocation between leisure trips and other goods. Namely, the consumer's utility maximization problem includes at least the triple constraints: the budget, time of weekend, and time of long holiday. This study develops a behavioral and econometric model of the leisure trip decision with these triple constraints.

As leisure time is a scarce resource, people value it highly. After Becker's (1965) seminal work, the value of time is a primary interest in the transport economics literature[1]. Jara-Díaz (2007) reviews theoretical models of the value of travel time savings. Hensher and Hotchkiss (1974) introduce the discrete choice model to estimate the value of time. Train and McFadden (1978) propose a discrete choice approach to model good and leisure allocation. Small and Verhoef (2007) review the empirical studies of the value of time in transportation. Kitamura (1984) considers the time allocation model between in-home and out-of-home activities. Bhat and Misra (1999) propose a time allocation model for weekdays and weekends.

Environmental economists have also discussed the value of leisure time and recreation demand[2]. Bockstael et al. (1989) review early studies on the role of time in the recreation demand model. Feather and Shaw (1999) estimate the value of time using stated preference data of labor choice and apply the results to estimate the demand for river recreation. Larson and Shaikh (2004) analyze the relationship between the value of time and the individual's wage using trip data of whale watching. Lew and Larson (2005) estimate the value of leisure time and beach recreation demand. Larson and Lew (2013) analyze the opportunity cost of travel

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[1] Early research on the value of time includes Becker (1965), DeSerpa (1971), Johnson (1966), Oort (1969), and Evans (1972).

[2] See Phaneuf and Smith (2005) for details of recreation demand models.

time in relation to salmon fishing. Fezzi et al. (2014) estimate the value of travel time using the driving route choice between open access and toll roads. However, most empirical studies of recreation demand assume that the value of leisure time is a fixed fraction of wages, following Cesario (1976), who suggests a rate of one-third of wages.

The consumer's leisure decision has multiple discrete-continuous choice characteristics. Namely, the individual may visit more than one site (multiple discrete choice), and the sites may be visited once or many times (continuous choice). In other words, leisure trips are characterized by a mixture of corner and interior solutions. For a given individual, some sites are unvisited (corner solution), whereas other sites are visited once or more (interior solution). To deal with this property of leisure trips, the Kuhn-Tucker (KT) demand model[3] and the multiple discrete–continuous extreme value (MDCEV) choice model (Bhat, 2005; 2008) were proposed. Both models use the Karush-Kuhn-Tucker utility maximization condition to estimate the parameters of the utility function. The difference between these two models is the assumption that the outside good is deterministic or random (Bhat, 2008). For the KT model, the outside good (the Hicks composite good) is assumed to be deterministic. For the MDCEV model, all goods are assumed to have error components with an extreme value distribution. As Bhat (2008) shows, the MDCEV is a general model that includes a conditional logit model and a KT model for special cases. LaMondia et al. (2008) apply the MDCEV model to estimate the value of leisure time. Castro et al. (2012) propose the MDCEV model with budget and time constraints to estimate the value of time[4].

This study proposes the MDCEV model with triple constraints to identify the value of leisure times for weekends and long holidays. Our proposed approach has two methodological contributions to the literature. First, we develop a conceptual and econometric model of the leisure trip decision with the triple constraints of

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[3] The KT demand model was proposed by Hanemann (1978) and Wales and Woodland (1983), and refined by Bockstael et al. (1989). Phaneuf et al. (2000) apply this model to estimate recreation demand. See von Haefen and Phaneuf (2005) for details of the KT demand model.

[4] While Castro et al. (2012) considered the MDCEV model with three or more constraints, they applied the MDCEV model with double constraints for their empirical study.

budget, duration of weekend, and duration of long holiday. We extend the MDCEV model to construct an estimation using the observed budget allocation of goods and time allocation between a weekend and a long holiday. Our proposed approach models endogenous time allocation and analyzes the substitution effect between weekends and long holidays. Second, we propose a numerical method of solving a system of nonlinear equations using the Karush-Kuhn-Tucker condition and the Markov Chain Monte Carlo (MCMC) method, which constructs the value of leisure time, demand prediction, and welfare analysis. To illustrate our proposed model, we apply it to recreation demand for national parks. The results suggest that a significantly large difference in the value of leisure time between weekends and long holidays.

Model

Model Structure

An individual is assumed to maximize utility subject to budget and time constraints. The time constraints include the time consumption of weekends and long vacations. The maximization problem for individual *n* is as follows:

$$
\begin{aligned}
&\max_{\mathbf{x_{nS}},\mathbf{x_{nL}}} U(\mathbf{x_{nS}}, \mathbf{x_{nL}}, \mathbf{z_n}, \mathbf{Q_{nS}}, \mathbf{Q_{nL}}, \boldsymbol{\varepsilon}_{\mathbf{nS}}, \boldsymbol{\varepsilon}_{\mathbf{nL}}, \boldsymbol{\varepsilon}_{\mathbf{nz}}; \boldsymbol{\beta}) \\
&\text{s.t. } z_{n1} + \sum_{k=1}^{K} (p_{nSk} x_{nSk} + p_{nLk} x_{nLk}) = E_n, \\
&\quad z_{n2} + \sum_{k=1}^{K} t_{nSk} x_{nSk} = T_{nS}, \\
&\quad z_{n3} + \sum_{k=1}^{K} t_{nLk} x_{nLk} = T_{nL}, \\
&\quad x_{nSk} \geq 0, x_{nLk} \geq 0, z_{n1} > 0, z_{n2} > 0, z_{n3} > 0,
\end{aligned}
\tag{1}
$$

where U is a quasi-concave, increasing, and continuously differentiable function, $\mathbf{x_{nS}} = (x_{nS1}, \cdots, x_{nSK})'$ is a vector of the consumption of K goods for weekend trips, $\mathbf{x_{nL}} = (x_{nL1}, \cdots, x_{nLK})'$ is a vector of the consumption of K goods for long vacation trips, $\mathbf{z_n} = (z_{n1}, z_{n2}, z_{n3})'$ is a vector of the consumption of numeraire goods, $\mathbf{Q_{nS}} = (\mathbf{q_{nS1}}, \cdots, \mathbf{q_{nSK}})'$ is a $K \times M$ matrix of M quality and individual attributes for the K goods for weekend trips, $\mathbf{Q_{nL}} = (\mathbf{q_{nL1}}, \cdots, \mathbf{q_{nLK}})'$ is a $K \times M$ matrix of M quality and individual attributes for the K goods for long vacation trips, $\boldsymbol{\varepsilon}_{\mathbf{nS}} = (\varepsilon_{nS1}, \cdots, \varepsilon_{nSK})'$ is a vector of random components for weekend trips, $\boldsymbol{\varepsilon}_{\mathbf{nL}} = (\varepsilon_{nL1}, \cdots, \varepsilon_{nLK})'$ is a vector of random components for weekend trips, $\boldsymbol{\varepsilon}_{\mathbf{nz}} = (\varepsilon_{nz1}, \varepsilon_{nz2}, \varepsilon_{nz3})'$ is a vector of random components for numeraire goods, $\boldsymbol{\beta}$ is a vector of parameters, $\mathbf{p_{nS}} = (p_{nS1}, \cdots, p_{nSK})'$ is a vector of the price for weekend trips, $\mathbf{p_{nL}} = (p_{nL1}, \cdots, p_{nLK})'$ is a vector of the price for long vacation trips, $\mathbf{t_{nS}} = (t_{nS1}, \cdots, t_{nSK})'$ is a vector of the two-way trip time for weekend trips, $\mathbf{t_{nL}} = (t_{nL1}, \cdots, t_{nLK})'$ is a vector of the two-way trip time for long vacation trips, E_n is the income, T_{nS} is the total time of weekends, and T_{nL} is the total time of long vacations.

We consider the case of three outside goods: z_{n1}, z_{n2}, and z_{n3}. The consumption of the outside goods is assumed strictly positive. The first good (z_{n1}) is a numeraire good with respect to the budget constraint. The second good (z_{n2}) is a numeraire good with respect to the time constraint of weekends. The third good (z_{n3}) is a numeraire good with respect to the time constraint of long vacations.

The Lagrangian function of the utility maximization problem is as follows:

$$\mathcal{L} = U(\mathbf{x}, \mathbf{z}, \mathbf{Q}, \boldsymbol{\varepsilon}_{\mathbf{x}}, \boldsymbol{\varepsilon}_{\mathbf{z}}; \boldsymbol{\beta}) + \lambda_1[E - z_1 - \mathbf{p}\mathbf{x}'] + \lambda_2[T_S - z_2 - \mathbf{t_S}\mathbf{x_S}'] + \lambda_3[T_L - z_3 - \mathbf{t_L}\mathbf{x_L}'], \qquad (2)$$

where $\mathbf{x} = (x_{S1}, \cdots, x_{SK}, x_{L1}, \cdots, x_{LK})' = (x_1, \cdots, x_{2K})'$, $\boldsymbol{\varepsilon}_{\mathbf{x}} = (\varepsilon_{S1}, \cdots, \varepsilon_{SK}, \varepsilon_{L1}, \cdots, \varepsilon_{LK})' = (\varepsilon_1, \cdots, \varepsilon_{2K})'$, $\mathbf{p} =$ $= (p_{S1}, \cdots, p_{SK}, p_{L1}, \cdots, p_{LK})' = (p_1, \cdots, p_{2K})'$, and $\mathbf{t} = (t_{S1}, \cdots, t_{SK}, t_{L1}, \cdots, t_{LK})' = (t_1, \cdots, t_{2K})'$ are $2K \times 1$ vectors for weekend and long vacation trips. $\mathbf{Q} = (\mathbf{q_{S1}}, \cdots, \mathbf{q_{SK}}, \mathbf{q_{L1}}, \cdots, \mathbf{q_{LK}})' = (\mathbf{q_1}, \cdots, \mathbf{q_{2K}})'$ is a $2K \times M$ matrix of quality and individual attributes, λ_1, λ_2, and λ_3 are the Lagrangian multipliers for each constraint and the index n is omitted for simplification. The first-order condition of the utility maximization problem can be written as

$$
\begin{aligned}
&\frac{\partial U(\mathbf{x}^*, \mathbf{z}^*, \mathbf{Q}, \boldsymbol{\varepsilon}_{\mathbf{x}}, \boldsymbol{\varepsilon}_{\mathbf{z}}; \boldsymbol{\beta})}{\partial x_k} < p_k\lambda_1 + s_k t_k \lambda_2 + [1 - s_k]\cdot t_k\lambda_3 \ \text{ if } x_k = 0, k = 1, \cdots, 2K, \\
&\frac{\partial U(\mathbf{x}^*, \mathbf{z}^*, \mathbf{Q}, \boldsymbol{\varepsilon}_{\mathbf{x}}, \boldsymbol{\varepsilon}_{\mathbf{z}}; \boldsymbol{\beta})}{\partial x_k} = p_k\lambda_1 + s_k t_k \lambda_2 + [1 - s_k]\cdot t_k\lambda_3 \ \text{ if } x_k > 0, k = 1, \cdots, 2K, \\
&\frac{\partial U}{\partial z_1} = \lambda_1, \frac{\partial U}{\partial z_2} = \lambda_2, \frac{\partial U}{\partial z_3} = \lambda_3,
\end{aligned}
\tag{3}
$$

where $\mathbf{x}^* = (x_{1S}^*, \cdots, x_{KS}^*, x_{1L}^*, \cdots, x_{KL}^*)' = (x_1^*, \cdots, x_{2K}^*)'$ and $\mathbf{z}^* = (z_1^*, z_2^*, z_3^*)$ are vectors of optimal consumption, s_k is the dummy for a weekend trip. Namely, $s_k = 1, \forall k \le K$ and $s_k = 0, \forall k > K$. Following Phaneuf et al. (2000) and Bhat (2008), we assume an additive separable utility function such that

$$
\begin{aligned}
U(\mathbf{x}, \mathbf{z}, \mathbf{Q}, \boldsymbol{\varepsilon}_{\mathbf{x}}, \boldsymbol{\varepsilon}_{\mathbf{z}}; \boldsymbol{\beta}) &= \sum_{m=1}^{3} u_{zm}(z_m, \varepsilon_{zm}; \boldsymbol{\beta}) + \sum_{k=1}^{2K} u_{xk}(x_k, \mathbf{q_k}, \varepsilon_{xk}; \boldsymbol{\beta}), \\
&\frac{\partial u_{zm}}{\partial \varepsilon_{zm}} > 0 (\forall m = 1,2,3), \frac{\partial u_{xk}}{\partial \varepsilon_{xk}} > 0 \ (\forall k = 1, \cdots, 2K).
\end{aligned}
\tag{4}
$$

Note that the utility function has two components, one for outside goods (u_{zm}) and the other for inside goods (u_{xk}). The inside goods component consists of the utilities for the weekend trip ($k \le K$) and the long vacation trip ($k > K$).

Then, the first-order Karush-Kuhn-Tucker condition can be rewritten as

$$
\begin{aligned}
&\varepsilon_{xk} < W_k(\boldsymbol{\varepsilon}_{\mathbf{z}}, x_k^*, \mathbf{z}^*, \mathbf{q_k}; \boldsymbol{\beta}) \ \text{ if } x_k^* = 0, k = 1, \cdots 2K, \\
&\varepsilon_{xk} = W_k(\boldsymbol{\varepsilon}_{\mathbf{z}}, x_k^*, \mathbf{z}^*, \mathbf{q_k}; \boldsymbol{\beta}) \ \text{ if } x_k^* > 0, k = 1, \cdots 2K,
\end{aligned}
\tag{5}
$$

where W_k is the solution to

$$\frac{\partial u_{xk}(x_k^*, \mathbf{q_k}, W_k; \boldsymbol{\beta})}{\partial x_k} = p_k \frac{\partial u_{z1}(z_1^*, \varepsilon_{z1}; \boldsymbol{\beta})}{\partial z_1} + s_k t_k \frac{\partial u_{z2}(z_2^*, \varepsilon_{z2}; \boldsymbol{\beta})}{\partial z_2} + [1 - s_k] t_k \frac{\partial u_{z3}(z_3^*, \varepsilon_{z3}; \boldsymbol{\beta})}{\partial z_3}. \tag{6}$$

Assume that $\boldsymbol{\varepsilon}_{\mathbf{x}}$ for inside goods is an independent and identically distributed draw from the type-I extreme value distribution with the inverse scale parameter σ_{xk}. The probability of observed consumption conditional on the random components of outside goods $\boldsymbol{\varepsilon}_{\mathbf{z}}$ is

$$P(\mathbf{x}^*, \mathbf{z}^* | \boldsymbol{\varepsilon}_{\mathbf{z}}, \mathbf{Q}, \boldsymbol{\beta}) = |J| \prod_{k=1}^{2K} \left[\frac{1}{\sigma_{xk}} \exp\left(-\frac{W_k}{\sigma_{xk}} \right) \right]^{1[x_k^* > 0]} \exp\left(-\exp\left(-\frac{W_k}{\sigma_{xk}} \right) \right), \tag{7}$$

where $|J|$ is the determinant of the Jacobian for the transformation and $1[x_k^* > 0]$ is an indicator function equal to one if x_k^* is strictly positive and zero otherwise. The unconditional probability is

$$P(\mathbf{x}^*, \mathbf{z}^* | \mathbf{Q}, \boldsymbol{\beta}) = \int_{\varepsilon_{z1}=-\infty}^{\infty} \int_{\varepsilon_{z2}=-\infty}^{\infty} \int_{\varepsilon_{z3}=-\infty}^{\infty} P(\mathbf{x}^*, \mathbf{z}^* | \boldsymbol{\varepsilon}_{\mathbf{z}}, \mathbf{Q}, \boldsymbol{\beta}) g_1(\boldsymbol{\varepsilon}_{\mathbf{z}}) d\varepsilon_{z3} d\varepsilon_{z2} d\varepsilon_{z1}, \tag{8}$$

where $g_1(\boldsymbol{\varepsilon}_{\mathbf{z}})$ is the density function of the random components for the outside goods. As there is no closed form of this probability for our preference specification (described later), it is necessary to approximate using a simulation (Train, 2009). The simulated probability is

$$\breve{P}(\mathbf{x}^*, \mathbf{z}^* | \mathbf{Q}, \boldsymbol{\beta}) = \frac{1}{R} \sum_{r=1}^{R} P(\mathbf{x}^*, \mathbf{z}^* | \boldsymbol{\varepsilon}_{\mathbf{z}}^r, \mathbf{Q}, \boldsymbol{\beta}), \tag{9}$$

where *R* is the number of draws and $\boldsymbol{\varepsilon}_{\mathbf{z}}^r$ is the *r*th draw from density function $g_1(\boldsymbol{\varepsilon}_{\mathbf{z}})$. The parameters of the simulated probability of the observed consumption $\check{P}(\mathbf{x}^*, \mathbf{z}^*)$ can be estimated by maximizing the simulated log-likelihood:

$$SLL = \sum_n \ln \check{P}(\mathbf{x_n^*}, \mathbf{z_n^*} | \mathbf{Q}, \boldsymbol{\beta}). \tag{10}$$

Preference specification

Following Bhat (2008) and Castro et al. (2012), we consider the following utility function:

$$\begin{aligned} U(x) &= \sum_{m=1}^{3} \frac{\psi_{zm}}{\alpha_{zm}} z_m^{\alpha_{zm}} + \sum_{k=1}^{2K} \frac{\gamma_{xk}\psi_{xk}}{\alpha_{xk}} \left[\left(\frac{x_k}{\gamma_{xk}} + 1 \right)^{\alpha_{xk}} - 1 \right], \\ \psi_{zm} &= \exp(\boldsymbol{\delta}'_{\mathbf{zm}} \mathbf{q_{zm}} + \varepsilon_{zm}), \alpha_{zm} = 1 - \exp(\alpha^*_{zm}), \forall m = 1,2,3, \\ \psi_{xk} &= \exp(\boldsymbol{\delta}'_{\mathbf{xk}} \mathbf{q_{xk}} + \varepsilon_{xk}), \alpha_{xk} = 1 - \exp(\alpha^*_{xk}), \gamma_{xk} = \exp(\gamma^*_{xk}) \ \forall k = 1, \cdots, 2K, \end{aligned} \tag{11}$$

where α^*_{zm}, α^*_{xk}, $\boldsymbol{\delta}_{\mathbf{zm}}$, $\boldsymbol{\delta}_{\mathbf{xk}}$, and γ^*_{xk} are estimated parameters. Note that the proposed function in (11) is a valid utility function for all parameter values, because the well-defined condition ($\psi_{zm}, \psi_{xk} > 0$, $\alpha_{zm}, \alpha_{xk} \leq 1$, $\gamma_{xk} > 0$) and weak complementarity for $\mathbf{q_{xk}}$ is satisfied for all parameter values[5]. The first-order Karush-Kuhn-Tucker condition of the utility maximization is

[5] Weak complementarity implies that the attributes of good k do not matter unless good k is actually consumed ($\partial U / \partial q_k = 0$ if $x_k = 0, \forall k$). For more on weak complementarity, see Mäler (1974), Hanemann (1984), and Herriges et al. (2004).

$$
\begin{aligned}
&\psi_{xk}\left(\frac{x_k}{\gamma_{xk}}+1\right)^{\alpha_{xk}-1} < p_k\psi_{z1}z_1^{\alpha_{z1}-1}+s_kt_k\psi_{z2}z_2^{\alpha_{z2}-1} \\
&\qquad\qquad +[1-s_k]t_k\psi_{z3}z_3^{\alpha_{z3}-1} \qquad \text{if } x_k=0, k=1,\cdots 2K, \\
&\psi_{xk}\left(\frac{x_k}{\gamma_{xk}}+1\right)^{\alpha_{xk}-1} = p_k\psi_{z1}z_1^{\alpha_{z1}-1}+s_kt_k\psi_{z2}z_2^{\alpha_{z2}-1} \\
&\qquad\qquad +[1-s_k]t_k\psi_{z3}z_3^{\alpha_{z3}-1} \qquad \text{if } x_k>0, k=1,\cdots 2K, \\
&\psi_{z1}z_1^{\alpha_{z1}-1}=\lambda_1, \psi_{z2}z_2^{\alpha_{z2}-1}=\lambda_2, \psi_{z3}z_3^{\alpha_{z3}-1}=\lambda_3.
\end{aligned} \tag{12}
$$

Then, W_k in (5) can be rewritten as

$$
\begin{aligned}
&W_k(\mathbf{x}^*,\mathbf{z}^*,\boldsymbol{\varepsilon}_\mathbf{z},\mathbf{q}_\mathbf{k},\boldsymbol{\beta}) \\
&\qquad = \ln\left\{p_k\psi_{z1}z_1^{\alpha_{z1}-1}+s_kt_k\psi_{z2}z_2^{\alpha_{z2}-1}+ [1-s_k]t_k\psi_{z3}z_3^{\alpha_{z3}-1}\right\} \\
&\qquad + (1-\alpha_{xk})\ln\left(\frac{x_k}{\gamma_{xk}}+1\right)-\boldsymbol{\delta}'_\mathbf{xk}\mathbf{q}_\mathbf{xk}
\end{aligned} \tag{13}
$$

The Jacobian for the transformation from ε_{xj} to x_k in (7) is

$$
J=\left[\frac{\partial\varepsilon_{xj}}{\partial x_k}\right], \tag{14}
$$

where

$$
\begin{aligned}
&\frac{\partial\varepsilon_{xj}}{\partial x_k}=\frac{A_{jk}+B_{jk}+C_{jk}}{D_j}\cdot 1\left[x_j\neq 0\right] \text{ for } \forall j\neq k, \\
&\frac{\partial\varepsilon_{xk}}{\partial x_k}=\left(\frac{A_{kk}+B_{kk}+C_{kk}}{D_k}+\frac{1-\alpha_{xk}}{x_k+\gamma_{xk}}\right)\cdot 1[x_k\neq 0]+1[x_k=0] \text{ for } \forall k, \\
&A_{jk}=p_jp_k\psi_{z1}(1-\alpha_{z1})z_1^{\alpha_{z1}-2}, \\
&B_{jk}=t_jt_k\cdot s_js_k\psi_{z2}(1-\alpha_{z2})z_2^{\alpha_{z2}-2}, \\
&C_{jk}=t_jt_k\cdot\left[1-s_j\right]\cdot[1-s_k]\psi_{z3}(1-\alpha_{z3})z_3^{\alpha_{z3}-2},
\end{aligned} \tag{15}
$$

$$D_k = p_k \psi_{z1} x_1^{\alpha_{z1}-1} + s_k t_k \psi_{z2} x_2^{\alpha_{z2}-1} + [1 - s_k] t_k \psi_{z3} x_3^{\alpha_{z3}-1}.$$

Our proposed model is the MDCEV model with three constraints, which includes the MDCEV models with single and double constraints. If $\psi_{z2} = 0$, $\psi_{z3} = 0$ and $\psi_{xk} = 0 \; \forall k > K$, it becomes the MDCEV model with a single constraint (Bhat, 2008). If $\psi_{z2} = 0$, $\psi_{z3} = 0$, $\varepsilon_{z1} = 0$, $\alpha_{xk} \to 0 \; \forall k$ and $\psi_{xk} = 0 \; \forall k > K$, it is a similar model to the Kuhn-Tucker with a single constraint (Phaneuf et al., 2000; von Haefen et al., 2004). If $\psi_{z3} = 0$ and $\psi_{xk} = 0 \; \forall k > K$, the model becomes the MDCEV model with double constraints (Castro et al., 2012). In addition, as shown in Castro et al. (2012), the model includes Satomura et al.'s (2011) double constraints model.

Solving Demand and the Value of Time

From the first-order condition (12), the optimal consumption of good *k* conditional on $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_{\mathbf{z}}' \; \boldsymbol{\varepsilon}_{\mathbf{x}}']'$ is

$$x_k(\boldsymbol{\varepsilon}) = Max\left[\gamma_{xk}\left(\frac{\psi_{xk}}{\lambda_1 p_k + s_k \lambda_2 t_k + [1 - s_k]\lambda_3 t_k}\right)^{\frac{1}{1-\alpha_{xk}}} - \gamma_{xk}, 0\right]. \tag{16}$$

Note that only x_k and $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ enter in (16) for the proposed additive separable utility function. This suggests that solving for the optimal consumption x_k reduces solving for $\boldsymbol{\lambda}$. The budget and time constraints can be written as

$$f_1(\lambda_1 | \boldsymbol{\varepsilon}) = \sum_{k=1}^{2K} (p_k x_k^*) + \left(\frac{\psi_{z1}}{\lambda_1}\right)^{\frac{1}{1-\alpha_{z1}}} - E = 0, \tag{17}$$

$$f_2(\lambda_2|\boldsymbol{\varepsilon}) = \sum_{k=1}^{2K}(s_k t_k x_k^*) + \left(\frac{\psi_{z2}}{\lambda_2}\right)^{\frac{1}{1-\alpha_{z2}}} - T_S = 0,$$

$$f_3(\lambda_3|\boldsymbol{\varepsilon}) = \sum_{k=1}^{2K}([1 - s_k]t_k x_k^*) + \left(\frac{\psi_{z3}}{\lambda_3}\right)^{\frac{1}{1-\alpha_{z3}}} - T_L = 0,$$

where x_k^* is the uncompensated demand for good k.

To solve for $\boldsymbol{\lambda}$, analysts could use the bisection algorithm (von Haefen et al., 2004; Lloyd-Smith, 2018). However, solving for three variables requires the triple nested bisection algorithm, making the calculation burden hard to ignore. For example, assume that solving one variable with the bisection algorithm uses 25 iterations. Solving three variables using the triple nested bisection algorithm would then require $25^3 = 15{,}625$ iterations. Pinjari and Bhat (2011) propose the analytic solution under the assumption that all satiation parameters are equal ($\alpha_k = \alpha$). However, their approach is inappropriate for our study because all satiation parameters are not equal. Alternatively, we use the numerical method of solving a system of nonlinear equations. Let $\boldsymbol{\lambda}^{\mathrm{i}}$ be a vector of three Lagrangian multipliers at iteration *m*. Under the numerical method, $\boldsymbol{\lambda}^{\mathrm{i}}$ can be updated using the following equation:

$$\boldsymbol{\lambda}^{\mathrm{i}+1} = \boldsymbol{\lambda}^{\mathrm{i}} - \mathbf{J_f}(\boldsymbol{\lambda})^{-1}\mathbf{f}(\boldsymbol{\lambda}), \tag{18}$$

where $\mathbf{J_f}(\boldsymbol{\lambda})$ is a Jacobian matrix of $\mathbf{f}(\boldsymbol{\lambda}) = [f_1(\lambda), f_2(\lambda), f_3(\lambda)]$. Therefore, to solve the consumer's utility maximization problem conditional on the random components $\boldsymbol{\varepsilon}$**,** we use the following numerical process :

1. Set the initial values $\boldsymbol{\lambda}^{\mathbf{0}}$:

$$\lambda_1^0 = \psi_{z1}\left(E - \sum_{k=1}^{2K} p_k x_k^*\right)^{\alpha_{z1}-1}, \tag{19}$$

$$\lambda_2^0 = \psi_{z2}\left(T_S - \sum_{k=1}^{2K} s_k t_k x_k^*\right)^{\alpha_{z2}-1},$$

$$\lambda_3^0 = \psi_{z3}\left(T_L - \sum_{k=1}^{2K} [1 - s_k] t_k x_k^*\right)^{\alpha_{z3}-1}.$$

2. At iteration *i*, conditional on $\boldsymbol{\lambda}^{\mathbf{i}}$ and $\boldsymbol{\varepsilon}$, calculate $\mathbf{x}^{\mathbf{i}}$ using (16).
3. Set the system of nonlinear equations using (17).
4. Update $\boldsymbol{\lambda}^{\mathbf{i+1}}$ using (18).
5. Iterate these steps until convergence.

This algorithm solves $\boldsymbol{\lambda}^*(\boldsymbol{\varepsilon})$ and the uncompensated demand $\mathbf{x}^*(\boldsymbol{\varepsilon})$ conditional on the random components. The values of leisure time of a weekend (VLT_S) and long vacation (VLT_L) are defined as follows:

$$VLT_S(\boldsymbol{\varepsilon}) = \frac{\lambda_2^*(\boldsymbol{\varepsilon})}{\lambda_1^*(\boldsymbol{\varepsilon})}, VLT_L(\boldsymbol{\varepsilon}) = \frac{\lambda_3^*(\boldsymbol{\varepsilon})}{\lambda_1^*(\boldsymbol{\varepsilon})}. \tag{20}$$

For the welfare analysis, this algorithm can be applied to solve the consumer's expenditure minimization problem subject to the following utility and time constraints:

$$\begin{aligned}
f_1(\lambda_1|\boldsymbol{\varepsilon}) &= \sum_{m=1}^{3} \frac{\psi_{zm}}{\alpha_{zm}}\left(\frac{\psi_{zm}}{\lambda_m}\right)^{\frac{\alpha_{zm}}{1-\alpha_{zm}}} + \sum_{k=1}^{2K} \frac{\gamma_{xk}\psi_{xk}}{\alpha_{xk}}\left[\left(\frac{x_k^H}{\gamma_{xk}} + 1\right)^{\alpha_{xk}} - 1\right] - U^0 = 0, \\
f_2(\lambda_2|\boldsymbol{\varepsilon}) &= \sum_{k=1}^{2K} (s_k t_k x_k^H) + \left(\frac{\psi_{z2}}{\lambda_2}\right)^{\frac{1}{1-\alpha_{z2}}} - T_S = 0, \\
f_3(\lambda_3|\boldsymbol{\varepsilon}) &= \sum_{k=1}^{2K} ([1 - s_k] t_k x_k^H) + \left(\frac{\psi_{z3}}{\lambda_3}\right)^{\frac{1}{1-\alpha_{z3}}} - T_L = 0,
\end{aligned} \tag{21}$$

where U^0 is an initial utility level and x_k^H is the Hicks compensated demand for good *k*. The algorithm for solving the compensated demand $\mathbf{x}^{\mathbf{H}}(\boldsymbol{\varepsilon})$ is the same as that for uncompensated demand $\mathbf{x}^*(\boldsymbol{\varepsilon})$ except for the utility constraint $f_1(\boldsymbol{\lambda}|\boldsymbol{\varepsilon})$. The solution from the compensated demand can be used to construct the Hicksian compensating variation (CV).

$$CV(\boldsymbol{\varepsilon}) = E - z_1^H - \sum_{k=1}^{2K} p_k x_k^H. \tag{22}$$

Simulating Random Components with the Markov Chain Monte Carlo method

As there are random components in the solution for uncompensated demand $\mathbf{x}^*(\boldsymbol{\varepsilon})$, the value of leisure times $VLT_S(\boldsymbol{\varepsilon}), VLT_L(\boldsymbol{\varepsilon})$, and the compensating variation $CV(\boldsymbol{\varepsilon})$, it is impossible to get precise estimates. However, using a simulation, the analyst can calculate the expectation of uncompensated demand $\mathrm{E}[\mathbf{x}^*(\boldsymbol{\varepsilon})]$, the value of leisure times $\mathrm{E}[VLT_S(\boldsymbol{\varepsilon})], E[VLT_L(\boldsymbol{\varepsilon})]$, and the compensating variation $\mathrm{E}[CV(\boldsymbol{\varepsilon})]$.

As shown in von Haefen et al. (2004), the distribution of $\boldsymbol{\varepsilon}$ conditional on observed consumption $\mathbf{x}$ can be decomposed as follows:

$$g(\boldsymbol{\varepsilon}|\mathbf{x}) = g_1(\boldsymbol{\varepsilon}_{\mathbf{z}}|\mathbf{x})g_2(\boldsymbol{\varepsilon}_{\mathbf{x}}|\mathbf{x}), \tag{23}$$

where g_1 is the distribution of the random components for three outside goods conditional on observed consumption and g_2 is the distribution of the random components for other goods conditional on observed consumption. Assume that $\boldsymbol{\varepsilon}_{\mathbf{z}}$ is the draw from the type-I extreme value distribution with inverse scale parameter $\boldsymbol{\sigma}_{\mathbf{z}}$, while $\boldsymbol{\varepsilon}_{\mathbf{x}}$ is the draw from the type-I extreme value distribution with inverse scale parameter $\boldsymbol{\sigma}_{\mathbf{x}}$. Following von Haefen et al. (2004), we use an adaptive Metropolis-Hasting (MH) algorithm to simulate the random components of the outside goods from the distribution of $\boldsymbol{\varepsilon}$ conditional on observed consumption

$\mathbf{x}$, $g_1(\boldsymbol{\varepsilon}_\mathbf{z}|\mathbf{x})$. The steps of the MH algorithm are as follows:

1. Set the initial values $\varepsilon_{zk}^0 = 0$ for $k = 1,2,3$.
2. At iteration i, create trial values of $\tilde{\varepsilon}_{zk}^{i+1} = \varepsilon_{zk}^i + \rho^i \sigma_{zk} \eta^i$, where ρ^i is a scalar specified by the analyst, σ_{kz} is the inverse scale parameter, and η^i is the random draw from a standard normal density.
3. Construct the ratio

$$\chi^{i+1} = \frac{P\left(\mathbf{x}^* | \tilde{\varepsilon}_{z1}^{i+1}, \tilde{\varepsilon}_{z2}^{i+1}, \tilde{\varepsilon}_{z3}^{i+1}\right) g_1\left(\tilde{\varepsilon}_{z1}^{i+1}/\sigma_{z1}, \tilde{\varepsilon}_{z2}^{i+1}/\sigma_{z2}, \tilde{\varepsilon}_{z3}^{i+1}/\sigma_{z3}\right)}{P\left(\mathbf{x}^* | \tilde{\varepsilon}_{z1}^{i}, \tilde{\varepsilon}_{z2}^{i}, \tilde{\varepsilon}_{z3}^{i}\right) g_1\left(\tilde{\varepsilon}_{z1}^{i}/\sigma_{z1}, \tilde{\varepsilon}_{z2}^{i}/\sigma_{z2}, \tilde{\varepsilon}_{z3}^{i}/\sigma_{z3}\right)}, \tag{24}$$

   where $g_1(\cdot)$ is the density function of the type-I extreme value and $P(\cdot)$ is the probability defined in (7).
4. Draw μ^{i+1} from a standard uniform. If $\mu^{i+1} < \chi^{i+1}$, accept $\tilde{\boldsymbol{\varepsilon}}_\mathbf{z}^{\mathbf{i+1}}$ and set $\boldsymbol{\varepsilon}_\mathbf{z}^{\mathbf{i+1}} = \tilde{\boldsymbol{\varepsilon}}_\mathbf{z}^{\mathbf{i+1}}$. Otherwise, reject $\tilde{\boldsymbol{\varepsilon}}_\mathbf{z}^{\mathbf{i+1}}$ and set $\boldsymbol{\varepsilon}_\mathbf{z}^{\mathbf{i+1}} = \boldsymbol{\varepsilon}_\mathbf{z}^{\mathbf{i}}$.
5. Adjust ρ^{i+1}. The value of ρ^i is raised if the accept rate of candidate parameters is less than 0.3. Otherwise, it is lowered.
6. Iterate the process.

After a burn-in period, which is the iteration process before convergence, this MCMC sequence generates the draws from $g_1(\boldsymbol{\varepsilon}_\mathbf{z}|\mathbf{x})$. As the draws using the MH algorithm are correlated over iteration, we use every tenth draw after burn-in to construct the expected demand.

Finally, the draws from $g_2(\boldsymbol{\varepsilon}_\mathbf{x}|\mathbf{x})$ are constructed following von Haefen et al. (2004). If the consumption of good k is strictly positive ($x_k^* > 0$), the random component of good k is constructed by $\varepsilon_{xk}^i = W_k\left(\boldsymbol{\varepsilon}_\mathbf{z}^\mathbf{i}, \mathbf{z_k}; \boldsymbol{\beta}, s_k\right)$ defined in (5) and (13). Otherwise, $\varepsilon_{xk}^i \leq W_k$ can be simulated from the truncated type-I extreme value distribution:

$$\varepsilon_{xk}^{i} = -\ln(-\ln(\exp(-\exp(-W_k/\sigma_{xk}))\,\mu))\,\sigma_{xk} \;\; \forall k = 1, \cdots 2K, \tag{25}$$

where μ is a draw from a standard uniform. The generated draws $\boldsymbol{\varepsilon}^{\mathbf{i}}$ are used for simulating the uncompensated demand $\mathbf{x}^{*}(\boldsymbol{\varepsilon}^{\mathbf{i}})$, the value of leisure time of a weekend $VLT_S(\boldsymbol{\varepsilon}^{\mathbf{i}})$, the value of leisure time of a long holiday $VLT_L(\boldsymbol{\varepsilon}^{\mathbf{i}})$, the compensated demand $\mathbf{x}^{\mathbf{H}}(\boldsymbol{\varepsilon}^{\mathbf{i}})$, and the Hicksian compensating variation $CV(\boldsymbol{\varepsilon}^{\mathbf{i}})$. Averaging each of the simulated values over iterations provides the expected values of $\mathrm{E}[\mathbf{x}^{*}(\boldsymbol{\varepsilon})]$, $\mathrm{E}[VLT_S(\boldsymbol{\varepsilon})]$, $E[VLT_L(\boldsymbol{\varepsilon})]$, and $\mathrm{E}[CV(\boldsymbol{\varepsilon})]$.

Application

Data

To illustrate our proposed model, we applied it to recreational trips to national parks in Japan. There are 32 national parks in Japan, which include seven World Heritage sites. Each year, there are more than 354 million visitors to the national parks. Some national parks are located near metropolitan areas and can be accessed for a day trip on a weekend, while others are located far from cities and are difficult to visit as part of a one- or two-day trip. In Japan, the average number of holidays in 2015 was 121.9 days, which consists of weekends (97.3 days) and long vacations (24.6 days), including the New Year, spring and summer holidays, and paid vacation[6].

To analyze the recreation demand for national parks, we conducted a nationwide web survey in February 2016. Respondents were asked the number of visits to each national park in 2015, as well as whether their visits to each park were weekend trips[7]. The collected sample consisted of 2038 respondents with trip

[6] We use an average of the total number of holiday days estimated using survey data for Japanese workers (Intelligence, 2013) and Japanese travelers (forTravel, 2015).

[7] Respondents were asked to choose one from following six options: (1) usual weekends (Saturday and Sunday), (2) consecutive holidays, (3) spring and autumn long holidays, (4) new year and summer long holidays, (5) paid vacation, and (6) other. We define (1) as weekend trips and (2)–(6) as long vacation trips.

information for 3,370 trips, which consists of 1480 trips (43.9%) on weekends and 1890 trips (56.1%) on long holidays. Table 1 describes the trip distribution of our data[8]. Note that while 64% of respondents did not visit any national parks in 2015, 36% of respondents visited one or more sites.

We consider the 32 national parks as the choice set[9]. Table 2 reports the descriptive statistics of the data. *Price* is the two-way trip cost between the respondent's residence and each park, which includes airfare, railroad fare, expressway toll, and vehicle operation cost ($0.184 per mile)[10]. *Time* is the two-way trip time between the respondent's residence and each park. *Word heritage* is the dummy of the World Heritage sites. Vehicle access is regulated in some parks due to traffic and damage to the natural environment. *Vehicle regulation* is the dummy where private vehicle access to the park is prohibited.

Table 1

Table 2

3.2 Estimation Results

In our study, we compare four models: (1) the standard single-constraint MDCEV model (SC-MDCEV1), (2) the heteroscedastic single-constraint MDCEV model (SC-MDCEV2), (3) the double-constraints MDCEV model (DC-MDECV), and (4) the triple-constraints MDCEV model (TC-MDCEV), which is proposed in this study. For the SC-MDCEV1 and SC-MDCEV2, the individual is assumed to maximize the utility subject to the budget constraint only ($\psi_{z2} = \psi_{z3} = 0$). For the standard SC-MDCEV1, all random components are assumed to be draws from i.i.d. extreme values ($\sigma_{xk} = \sigma, \forall k$). For

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For example, three holidays—a weekend (Saturday, Sunday) and a paid vacation (Friday or Monday)—are defined as a long holiday.

[8] The trip distribution for each park is reported in Table 6.

[9] Two national parks, Yambaru and Amami-Gunto, were excluded from the choice set because they were established after our survey was conducted.

[10] Calculated using an average of gasoline prices, gas mileage, and exchange rate (US$ 1 = 121 yen). The remainder of the paper uses the same exchange rate.

the SC-MDCEV2, DC-MDCEV, and TC-MDCEV, the inverse scale parameters for outside goods are assumed different from those for inside goods ($\sigma_{zm} \neq \sigma,\ \sigma_{xk} = \sigma, \forall m \leq 3, \forall k \leq 2K$). For the DC-MDCEV, the consumer is assumed to maximize the utility subject to the budget and time constraints, but the time allocation between a weekend and long holiday is not considered ($\psi_{z3} = 0$). The time constraint is the yearly total of the average of all holidays ($T = T_S + T_L = 121.9$ days $\times$ 24h). For the TC-MDCEV, the individual is assumed to maximize utility subject to the budget, the weekend time, and the long holiday time constraints. The time constraint for a weekend is the yearly total of the average weekend time ($T_S = 97.3$ days $\times$ 24h) and the constraint for a long holiday is the yearly total of the average long holiday time ($T_L = 24.6$ days $\times$ 24h).

For the identification problem, it is difficult to estimate all parameters α_{xk} and γ_{xk} simultaneously (Bhat, 2008). For the Hicks composite good z_1, ψ_{z1} has only the random component and $\boldsymbol{\delta}$ is assumed to be zero due to the stability of estimation. Following Castro et al. (2012), we explored a number of alternative utility function forms that include the α-profile (setting the $\gamma_{xk} = 1$ and estimating α_{zk} and α_{xk}), γ-profile (setting the $\alpha_{zm} = \alpha_{xk} = 0$ and estimating γ_{xk}), and the $\alpha\gamma$-profile (a combination of the α-profile and γ-profile, namely, setting the $\gamma_{xk} = 1$ and estimating α_{zk} and α_{xk} for some goods and setting the $\alpha_{zm} = \alpha_{xk} = 0$ and estimating γ_{xk} for others). Finally, we adopted the following $\alpha\gamma$-profile function form:

$$
\begin{aligned}
&U(x) = \frac{\exp(\varepsilon_{z1})}{\alpha_{z1}} z_1^{\alpha_{z1}} + \exp(\xi_2)\ln z_2 + \exp(\xi_3)\ln z_3 + \sum_{k=1}^{2K} \gamma\psi_{xk} \ln\left(\frac{x_k}{\gamma} + 1\right), \\
&\psi_{xk} = \exp(\boldsymbol{\delta}'\mathbf{q_k} + \varepsilon_{xk}), \alpha_{z1} = 1 - \exp(\alpha_{z1}^*), \gamma = \exp(\gamma^*), \\
&\sigma_{z1} = \exp(\sigma_{z1}^*), \sigma_{xk} = \sigma = \exp(\sigma^*).
\end{aligned}
\tag{26}
$$

Note that this function form is a combination of the α-profile for the Hicks composite good z_1 and the γ-profile for other goods. The random components of the numeraire goods of time constraints are assumed to be zeros, since the inverse scale parameters (σ_{z2} and σ_{z3}) were not significant. One possible reason the error components of z_2 and z_3 are zeros is the small variation of z_2 and z_3. While the coefficient of variation of the Hicks composite good (z_1) is 0.595, the estimated coefficient of variation of z_2 and z_3 are 0.008 and 0.041, respectively.

Table 3 presents the estimated parameters of four models. The parameters of the standard SC-MDCEV1 are estimated using the maximum log likelihood, whereas the parameters of the SC-MDCEV2, DC-MDCEV, and TC-MDCEV models are estimated using the simulated maximum log likelihood with 200 Halton draws[11]. The convergence of the TC-MDCEV required about five hours using a standard PC (Intel Core i7, 3.1 GHz). All parameters for the variables of the four models have the same signs. The parameters of the site-specific attributes in ψ_{xk} have significant and expected signs. The impact of the variables *world heritage*, *visitor center*, and *hot spring* have positive signs, while *vehicle regulation* has a negative effect on the utility.

To evaluate fit for each model, the adjusted rho-bar ($\bar{\rho}^2$) estimator can be applied (Castro et al., 2012). The estimator is defined as $\bar{\rho}^2 = 1 - [(LL(\beta) - H + H_0)/LL_0]$, where $LL(\beta)$ is the log likelihood at convergence, LL_0 is the log likelihood at the base model, which is assumed to include the baseline constant in the ψ function and the translation parameter (γ) only, H is the number of parameters for the estimation models, and H_0 is the number of parameters for the base model. The adjusted rho-bar estimator in Table 3 suggests that our proposed TC-MDCEV model is the best fit for the data.

Table 3

For the welfare analysis, we consider two scenarios: (A) Adding 1,000 yen (US$ 8.7) to the entrance fee for all parks, and (B) closure of the Fuji-Hakone-Izu national park. In Japan, most national parks have no entrance fee. However, the entrance fee at national parks is a controversial topic, as climbers at Mount Fuji were requested to pay the 1,000 yen fee after Mount Fuji was included on the UNESCO World Heritage List.

[11] Bhat (2001) found that 100 Halton draws provided more precise results than 1,000 random draws.

Mount Fuji is the highest mountain peak in Japan, and about 300,000 climbers visit Mount Fuji each summer[12]. Mount Fuji has not erupted since 1707, and at present, the Japan Meteorological Agency has assigned Mount Fuji its lowest threat level for volcanic eruption. However, closure of the national park due to an eruption of Mount Fuji might have a large and negative impact on climbers, even if the risk of eruption is very low. Mount Hakone is also located in the park, and in May 2015, Owakudani, the valley beneath Mount Hakone, was closed due to volcanic activity.

The estimated compensating variations of the two scenarios using four models are reported in Table 4. Welfare estimates were generated with 4,500 draws using an adaptive Metropolis-Hastings algorithm to simulate unobserved random components consistent with the individual's observed choice[13]. The first 500 were discarded as burn-in, and every tenth iteration thereafter was used to construct the expected compensating variations. To solve the compensated and uncompensated demand, we applied the GAUSS eqSolve command to a system of nonlinear equations in (17) and (21)[14]. For our proposed TC-MDCEV model, it requires about 6 minutes to get the point estimates of the compensated and uncompensated demand and 20 hours to estimate the 95% confidence intervals calculated using the procedure reported by Krinsky and Robb (1986), based on 200 iterations.

For the entrance fee scenario, the welfare loss per person is about \$13 for all models. On the other hand, for the park closure scenario, there is significantly large difference in the welfare loss between the single constraint models (\$31.39 and \$31.85) and the multiple constraints models (\$111.98 and \$116.32). One possible reason for the difference is the substitution effects under the budget and time constraints. Assume that an individual visits Mount Fuji to experience the World Heritage site. If Mount Fuji closes, the individual

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[12] In 2015, the mountain trails on Mount Fuji were opened for only 75 days through the year, from July 1 to September 14.
[13] To confirm the stability, we compared the estimated values using different numbers of draws. For example, the error differences in the values of leisure time are less than 1.5% at 1500 draws and 0.5% at 2500 draws when compared to the values in Table 5 based on 4500 draws.
[14] GAUSS codes for all estimation and welfare calculations discussed in this paper are available from the author's web site.

might want to visit substitute parks with World Heritage sites. However, the average two-way trip time for other World Heritage sites is 18 hours, while the average two-way time for Fuji-Hakone-Izu national park is about 7 hours[15]. Thus, it might be difficult to visit other World Heritage sites because of the time constraints, even if the individual could afford the trip cost to visit the substitute sites.

Table 4

The estimated values of leisure time (*VLT*) are presented in Table 5. The values of leisure time for a weekend and a long holiday are defined in (20) and were estimated using an adaptive Metropolis-Hastings algorithm. The value of leisure time for all holidays estimated by the DC-MDCEV is \$8.08 / h, which is similar to the average value of travel time savings for leisure (\$8.48 / h) using a meta-analysis of 68 papers for trip data in Japan (Kato et al., 2010). From the estimation results of our proposed TC-MDCEV, the value of leisure time for a weekend is \$17.82 / h, while the one for a long holiday is \$8.36 / h. These results suggest a significantly large difference in the value of leisure time between a weekend and a long holiday.

One possible reason for this large difference is the great scarcity of time on weekends since visitors are limited to two-day trips on weekends. A trip to a national park is a time-consuming recreation, and some visitors tend to prefer other leisure-time activities than visiting a national park. In contrast, scarcity of time is relatively low in a long vacation, and visitors can spend three or more days for a trip to a national park.

Table 5

Finally, we investigate the effect of the time constraints on the trip distribution. Table 6 reports the average predicted trips to each park when the time constraints of a weekend and long holiday are increased by

[15] Table 6 reports the average two-way trip time to each park.

five days. Considering the time effects on the number of all holiday trips, the predicted trips to parks located near metropolitan areas (#10, #11, and #14) tend to increase with time extension on a weekend, while the number of trips to faraway parks (#2 and #15) is predicted to grow with time extension in a long holiday[16]. Note that it is difficult to visit the World Heritage site Ogasawara (#15) on a weekend, as the two-way trip time to the park is more than 56 hours[17]. Therefore, predicted trips to the site do not change with time extension on a weekend, whereas the number of trips is predicted to increase to 121.9% with time extension in a long holiday. Furthermore, Table 6 suggests that the substitution effects between weekend and long holiday trips are very small. The number of trips is predicted to increase to 104.6% with a weekend extension but remains almost unchanged in a long holiday. With a long holiday extension, however, the predicted increase in the number of trips is 115.0%, while the decrease is considered negligible. The importance of considering the budget and time constraints is also evident. For example, the southwesternmost #32 (Iriomote-Ishigaki) National Park, with the highest average price, shows a relatively small increase in both time extension scenarios. With the high travel cost, it might to be difficult for most visitors to increase their trips to this park because of the budget constraint, even if the time constraint is relaxed.

Table 6

Concluding comments

In this study, we propose the MDCEV model with the triple constraints of budget, length of weekend, and length of holiday. Our proposed model's methodological contribution is to capture the difference in leisure behavior between a weekend and a long holiday. We develop an estimation model of the utility function using the observed trip demand and time allocation over a weekend and a long holiday. Moreover, we develop a

[16] Note that #2 is the easternmost, and #15 is the southeasternmost national park.
[17] Access to the site is limited to ships, since there is no airport on the Ogasawara islands.

numerical method for solving a system of nonlinear equations using the Karush-Kuhn-Tucker condition, which constructs the value of leisure time, predicts demand, and analyzes welfare impacts. Our proposed algorithm addresses the numerical difficulties of the nested bisection algorithm proposed by previous studies.

In the empirical section, we apply the MDCEV model with triple constraints for leisure trips to national parks in Japan. The estimation results of the MDCEV models with single, double, and triple constraints are compared, and our proposed model shows the best fit for the data. For the welfare analysis, we consider the two scenarios of adding an entrance fee to all parks and the closure of a world heritage site. The welfare losses of the entrance fee scenario are slightly different for all models. However, for the park closure scenario, there is a significantly large difference in welfare loss between the single- and multiple-constraint models. Moreover, estimation results by the MDCEV model with triple constraints show a significantly large difference in the value of leisure time and trip prediction between a weekend and a long holiday. Estimation results show that the substitution effects between weekend and long holiday trips are very small. Some national parks face congestion problems in summer holidays, and the park management might consider distributed recreation use from summer holidays to weekends. However, our empirical results show that weekend trips cannot substitute for long holiday trips. Thus, visitor control, such as car regulation or higher entrance fee in summer holidays, will not shift recreation use from summer holidays to weekends, but merely decrease the number of visitors in summer holidays.

These results suggest that it is important to differentiate between weekend and long holiday leisure time. The value of time is one of the most important issues for recreation and transportation policy analysis, and the literature contains extensive empirical research on the value of time. Some studies have reported differences in the value of time between weekdays and weekends; however, the time value difference between weekends and long holidays has not been considered. Our findings have important implications for recreation and transportation policy analysis by showing the time value difference between weekends and long vacations.

Finally, we suggest two areas for future research. First, the heterogeneity of preferences for the

MDCEV model with triple constraints should be investigated. For the single constraint, several models are proposed to implement the heterogeneity, including the random parameter KT model (von Haefen and Phaneuf, 2005), the mixed MDCEV model (Bhat, 2008), the latent segmentation KT model (Kuriyama et al., 2010), and the latent segmentation MDCEV model (Wafa et al., 2015). The random parameter and latent segmentation approach to the MDCEV model with multiple constraints may be worth investigating. Second, the spatial econometric approach should be explored. The geographical attributes of recreation sites, such as altitude, temperature, vegetation, and land use, are distributed spatially. For the single constraint model, Kuriyama et al. (2012) apply the spatial KT model and Bhat et al. (2015) propose the spatial multiple discrete-continuous probit model. The spatial MDCEV model with multiple constraints may be an important extension for future research.

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Table 1 Trip Distribution

Number of trips taken per respondent	Number of observations	
0	1314	64%
1	261	13%
2	140	7%
3	90	4%
4	60	3%
5	36	2%
6	17	1%
7	22	1%
8	7	0%
9	9	0%
10 and over	82	4%
Total	2038	100%

Table 2 Descriptive Statistics

Variables	Description	mean	std. dev.	min	max
x	The number of trips per individual	1.654	4.764	0.000	70.000
Price	Two-way trip cost to each site (US$)	520.855	313.201	5.556	1646.515
Time	Two-way trip time to each site (hour)	11.350	9.300	0.900	70.500
Male	1/0, 1 = male	0.483	0.500	0.000	1.000
Income	Household income (US$)	54311	32135	16529	165289
World heritage	1/0, 1 = World Heritage site	0.219	0.413	0.000	1.000
Visitor center	The number of visitor centers	1.125	1.833	0.000	8.000
Hot spring	1/0, 1 = hot spring	0.375	0.484	0.000	1.000
Vehicle regulation	1/0, 1 = vehicle regulation	0.594	0.491	0.000	1.000

Table 3 Estimation Results

		Model 1 SC-MDCEV1		Model 2 SC-MDCEV2		Model 3 DC-MDCEV		Model 4 TC-MDCEV	
ψ_{xk}	Constant	-5.490	***	-1.4595		-3.1378	*	-4.6732	***
		(-13.82)		(-1.15)		(-1.95)		(-2.69)	
	Male	0.254	***	0.2899	***	0.2368	***	0.2463	***
		(3.61)		(3.03)		(3.41)		(3.55)	
	World heritage	1.196	***	1.1568	***	1.0038	***	0.977	***
		(17.88)		(18.09)		(18.13)		(16.98)	
	Visitor center	0.267	***	0.2576	***	0.2072	***	0.1962	***
		(14.30)		(14.40)		(13.89)		(12.65)	
	hot spring	1.170	***	1.1393	***	0.8084	***	0.7514	***
		(17.01)		(17.28)		(14.53)		(13.18)	
	Vehicle regulation	-1.029	***	-0.9489	***	-0.8218	***	-0.7888	***
		(-14.27)		(-13.55)		(-14.28)		(-13.25)	
ψ_{z2}	ξ_S					4.1373	***	3.6542	**
						(2.58)		(2.11)	
ψ_{z3}	ξ_L							1.4249	
								(0.83)	
α_{z1}^*		-1.0405	***	-0.7872	***	-0.4681	***	-0.3753	**
		(-6.18)		(-4.45)		(-2.83)		(-2.32)	
γ^*		0.2256	***	0.2368	***	0.4971	***	0.533	***
		(5.38)		(5.73)		(11.21)		(11.84)	
σ_{z1}^*		0.0156		0.6383	***	0.8089	***	0.8577	***
		(0.85)		(17.92)		(19.11)		(19.47)	
σ^*				-0.0409	**	-0.2526	***	-0.2077	***
				(-2.18)		(-10.45)		(-8.69)	
	LL	-8989.4		-8833.7		-8739.4		-9909.9	
	LL_0	-9335.9		-9335.9		-9331.0		-10829.3	
	$H - H_0$	7		8		9		10	
	$\bar{\rho}^2$	0.036		0.053		0.062		0.084	

Note. The values in parentheses are t-statistics. All parameters are estimated using 200 Halton draws except model 1.

** $p < .05$

*** $p < .01$

Table 4 Welfare Analysis

Policy scenarios	model 1 SC-MDCEV1	model 2 SC-MDCEV2	model 3 DC-MDCEV	model 4 TC-MDECV
Entrance fee of $8.30 in all parks	-12.750 [-12.8, -12.7]	-12.750 [-12.8, -12.7]	-13.124 [-13.2, -13.1]	-13.166 [-13.2, -13.1]
Closure of Fuji-Hakone-Izu National Park	-31.390 [-32.9, -30.0]	-31.854 [-33.3, -30.2]	-111.977 [-148.9, -82.7]	-116.317 [-220.5, -34.6]

Note. USD per person. Welfare estimates generated with 4,500 draws. The first 500 were discarded as burn-in, and every tenth iteration thereafter was used to construct the expected compensating variations. The values in brackets are 95% confidence intervals calculated using the procedure reported by Krinsky and Robb (1986), based on 200 iterations.

Table 5 The Value of Leisure Times (VLT)

	model 3 DC-MDCEV	model 4 TC-MDECV
VLT of all holiday	8.084 [6.2, 12.9]	
VLT of a weekend (VLT_S)		17.823 [14.0, 26.5]
VLT of a long holiday (VLT_L)		8.359 [6.3, 13.8]
$VLT_S - VLT_L$		9.464 [7.3, 14.5]

Note. USD per hour. Welfare estimates generated with 4,500 draws. The first 500 were discarded as burn-in, and every tenth iteration thereafter was used to construct the expected value of times. The values in brackets are 95% confidence intervals calculated using the procedure reported by Krinsky and Robb (1986) and based on 200 iterations.

Table 6 Current and Predicted Trip Demand

	National Parks	Price (US$)	Time (h)	Weekend (per person)			Long holiday (per person)			All holiday (per person)		
				Current demand	Weekend extension	Long holiday extension	Current demand	Weekend extension	Long holiday extension	Current demand	Weekend extension	Long holiday extension
1	Rishiri-Rebun-Sarobetsu	757	14.28	0.004	106.9%	99.8%	0.015	100.0%	116.8%	0.020	101.5%	111.3%
2	Shiretoko	771	15.22	0.004	106.7%	99.9%	0.020	99.9%	122.9%	0.025	101.1%	117.3%
3	Akan	744	10.25	0.005	106.9%	99.7%	0.013	99.9%	121.1%	0.018	102.0%	112.4%
4	Kushiroshitsugen	751	11.02	0.014	104.8%	99.7%	0.018	99.9%	121.6%	0.032	102.0%	109.9%
5	Daisetsuzan	736	12.12	0.016	104.2%	100.0%	0.025	99.9%	117.0%	0.041	101.6%	108.4%
6	Shikotsu-Toya	711	7.48	0.031	104.2%	99.9%	0.036	100.0%	111.1%	0.068	101.9%	103.9%
7	Towada-Hachimantai	559	15.73	0.018	105.8%	99.8%	0.033	100.0%	116.0%	0.051	102.0%	108.2%
8	Sanriku Fukko	532	16.23	0.012	106.2%	100.0%	0.017	100.0%	115.8%	0.029	102.6%	106.4%
9	Bandai-Asahi	395	9.55	0.005	105.4%	100.0%	0.013	100.0%	120.1%	0.018	101.4%	113.0%
10	Nikko	337	9.64	0.051	105.5%	99.9%	0.047	100.0%	118.3%	0.097	102.8%	105.7%
11	Oze	320	9.63	0.006	108.1%	99.9%	0.011	100.0%	124.6%	0.017	102.7%	113.0%
12	Myoko-Togakushi renzan	287	7.88	0.013	106.1%	99.8%	0.010	100.0%	122.5%	0.024	103.4%	106.1%
13	Joshin'etsukogen	287	7.88	0.021	104.7%	99.8%	0.021	100.0%	118.2%	0.042	102.3%	106.6%
14	Chichibu-Tama-Kai	258	7.15	0.026	105.4%	99.9%	0.026	100.0%	120.2%	0.052	102.6%	107.4%
15	Ogasawara	685	56.90	0.000	-	-	0.004	99.9%	121.9%	0.004	100.0%	121.7%
16	Fuji-Hakone-Izu	279	6.77	0.077	104.6%	99.9%	0.099	100.0%	113.0%	0.175	102.0%	105.2%
17	Chubusangaku	320	9.27	0.009	106.1%	99.8%	0.027	100.0%	115.3%	0.036	101.6%	109.6%
18	Minami Alps	278	7.19	0.016	105.5%	99.9%	0.027	100.0%	119.0%	0.043	102.0%	109.8%
19	Hakusan	321	9.29	0.007	105.8%	99.7%	0.010	100.0%	118.3%	0.018	102.4%	107.9%
20	Ise-Shima	325	9.30	0.057	104.3%	99.9%	0.076	100.0%	112.5%	0.133	101.9%	105.1%
21	Yoshino-Kumano	291	10.25	0.006	106.9%	99.9%	0.027	100.0%	121.8%	0.033	101.2%	116.4%
22	San'inkaigan	334	11.25	0.035	103.9%	99.9%	0.025	100.0%	120.1%	0.059	102.3%	105.8%
23	Setonaikai	373	11.06	0.143	103.4%	99.8%	0.129	100.0%	109.8%	0.272	101.8%	102.7%
24	Daisen-Oki	385	11.65	0.003	106.8%	99.9%	0.019	100.0%	119.4%	0.022	100.9%	115.6%
25	Ashizuri-Uwakai	581	9.55	0.019	105.3%	99.8%	0.003	99.9%	121.0%	0.022	104.6%	98.2%
26	Saikai	571	7.59	0.010	104.9%	99.9%	0.011	100.0%	113.7%	0.021	102.4%	104.5%
27	Unzen-Amakusa	580	8.46	0.024	103.6%	100.0%	0.037	100.0%	112.6%	0.061	101.4%	106.1%
28	Aso-Kuju	570	7.89	0.062	104.8%	100.0%	0.054	100.0%	113.0%	0.116	102.6%	103.4%
29	Kirishima-Kinkowan	594	9.32	0.018	104.6%	100.0%	0.026	100.0%	115.3%	0.045	101.9%	107.0%
30	Yakushima	853	6.74	0.002	105.4%	99.3%	0.013	99.9%	107.9%	0.015	100.6%	106.1%
31	Keramashoto	790	8.19	0.000	104.4%	100.0%	0.015	100.0%	113.5%	0.016	100.1%	112.9%
32	Iriomote-Ishigaki	1106	8.49	0.011	100.9%	100.0%	0.019	100.0%	104.5%	0.030	100.3%	102.5%
	Total	-	-	0.726	104.6%	99.9%	0.927	100.0%	115.0%	1.654	102.0%	106.2%

Note. Predicted demand generated with 4,500 draws. The first 500 were discarded as burn-in, and every tenth iteration thereafter was used to construct the expected demand.