

The Intertemporal Substitution of Recreation Demand:

A Dynamic Kuhn-Tucker Model with a Corner Solution

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Abstract The demand for recreation exhibits intertemporal substitution, as well as spatial substitution. In this paper, we develop a dynamic Kuhn-Tucker model of recreation demand with intertemporal substitution effects, from both theoretical and empirical perspectives. We propose a dynamic extension of the traditional static Kuhn-Tucker model and apply this model to data for the beaches of Southern California. The empirical results suggest that disregarding intertemporal substitution effects may result in an overstatement of welfare loss; however, the difference in the compensating variations of the dynamic models with and without intertemporal substitution effects is relatively small for our dataset.

KEY WORDS welfare analysis, demand system, dynamic optimization, beach recreation

JEL Classification: Q26

1 INTRODUCTION

The demand for recreation exhibits intertemporal substitution, as well as spatial substitution. For example, in the event of a beach closure, an individual may visit alternative beaches during a given time period or may postpone trips entirely until the closed site is reopened. Moreover, recreation demand exhibits a point mass near zero. While some visitors recreate year round, most individuals choose either no trips or a small number of trips in the summer months. Therefore, a dynamic behavior model that allows for a corner solution may be required for analyzing the intertemporal substitution of recreation demand.

The Kuhn-Tucker model has been proposed as a means of dealing with corner solutions. The Kuhn-Tucker model relies on a single structural framework that simultaneously incorporates both site selection and participation decisions with corner solutions. The Kuhn-Tucker model was initially developed by Wales and Woodland (1983) and Hanemann (1978), and it was refined by Bockstael, Hanemann, and Strand (1986). Phaneuf, Kling, and Herriges (2000, hereafter PKH) provided an application of this model to recreation demand. While Herriges, Kling, and Phaneuf (1999) compared the Kuhn-Tucker model with the linked model, von Haefen and Phaneuf (2003) compared this model with the count model. PKH's approach to the analytical calculation of compensating variation is infeasible when the size of the choice set is relatively large; however, von Haefen, Phaneuf, and Parsons (forthcoming, hereafter vHPP) have recently proposed a Markov Chain Monte Carlo (MCMC) algorithm for constructing compensating variations for large-choice-set data. Previous studies of the Kuhn-Tucker model have analyzed static utility maximization problems that assumed no intertemporal substitution.

In this paper, we develop a dynamic Kuhn-Tucker model of recreation demand with intertemporal substitution effects from both theoretical and empirical perspectives. We provide a dynamic extension of the traditional, static Kuhn-Tucker model. We then report

the results of an empirical application of our model to data for the beaches of Southern California; these results allow us to compare our proposed dynamic model and an analogous model without intertemporal substitution.

The remainder of the paper is structured as follows. In section 2, we provide an overview of the dynamic Kuhn-Tucker continuous-demand model. Section 3 discusses a numerical bisection algorithm for constructing welfare changes in the context of intertemporal substitution. Section 4 presents the results of our empirical analysis of beach recreation data for the Southern California region. Finally, section 5 provides concluding comments.

2 MODEL

Dynamic Model and Intertemporal Substitution

As discussed by PKH, the formulation of the Kuhn-Tucker model begins with the assumption that consumer preferences can be represented by a random utility function, which can be maximized subject to a budget constraint and nonnegativity constraints. While the traditional Kuhn-Tucker model is concerned with static consumption allocation problems, this model can be extended to problems of intertemporal, or dynamic, allocation. Let

- $t = 1, \dots, T$ be the set of time periods,
- $U_t(\mathbf{x}_t, \mathbf{Q}_t, z_t, \varepsilon_t)$ represent a utility function that is assumed to be a quasi-concave, increasing, and continuously differentiable function of (\mathbf{x}_t, z_t) ,
- $\mathbf{x}_t = (x_{1t}, \dots, x_{Mt})'$ be an M -dimension vector of control variables that represents consumption goods in period t ,
- $\mathbf{Q}_t = (\mathbf{q}_{1t}, \dots, \mathbf{q}_{Mt})'$ be an $M \times K$ matrix of quality attributes for the M goods in period t ,

- z_t represent the Hicksian composite good in period t , which is assumed to be a necessary good,
- $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \dots, \varepsilon_{Mt})'$ be a vector of random components that are assumed to be known to the individual but unknown to the analyst,
- y_t be a state variable that represents residual income in period t , where $y_0 = y$ is the initial income and $y_T = 0$ is terminal income,
- $y_{t-1} - y_t = \mathbf{p}'_t \mathbf{x}_t + z_t$ represent the budget constraint in period t , and
- $\mathbf{p}_t = (p_{1t}, \dots, p_{Mt})'$ be a vector of prices in period t .

Then, consider the following dynamic allocation problem:

$$\begin{aligned}
V &= \text{Max}_{\mathbf{x}_t, z_t, y_t} \sum_{t=1}^T \xi^{t-1} U_t(\mathbf{x}_t, \mathbf{Q}_t, z_t, \boldsymbol{\varepsilon}_t) \\
\text{s.t. } &y_{t-1} - y_t = \mathbf{p}'_t \mathbf{x}_t + z_t, \\
&y_0 = y, y_T = 0, \\
&x_{jt} \geq 0,
\end{aligned} \tag{1}$$

for $j = 1, \dots, M$ and $t = 1, \dots, T$, where ξ is the discount factor. Note that the individual can control the budget allocation in each period ($y_{t-1} - y_t$). Therefore, current demand may be affected by changes in the prices or qualities of other periods, unless the demand functions exhibit no income effects. Thus, this model can be used to deal with intertemporal substitution. In this case, the Lagrangian is given by

$$L = \sum_{t=1}^T \left\{ \xi^{t-1} U_t(\mathbf{x}_t, \mathbf{Q}_t, z_t, \boldsymbol{\varepsilon}_t) + \lambda_t (y_{t-1} - y_t - \mathbf{p}'_t \mathbf{x}_t - z_t) \right\} \tag{2}$$

where λ_t is a Lagrange multiplier representing the marginal utility of income. The first-order Kuhn-Tucker conditions of the utility maximization problem are as follows:

$$\xi^{t-1} U_{jt} - \lambda_t p_{jt} \leq 0, \quad (3a)$$

$$\xi^{t-1} U_{zt} - \lambda_t = 0, \quad (3b)$$

$$x_{jt} \geq 0, \quad (3c)$$

$$x_{jt} [\xi^{t-1} U_{jt} - \lambda_t p_{jt}] = 0, \quad (3d)$$

$$\lambda_{t+1} = \lambda_t \equiv \lambda, \quad (3e)$$

$$y_{t-1} - y_t = \mathbf{p}_t' \mathbf{x}_t + z_t, \quad (3f)$$

for $j = 1, \dots, M$ and $t = 1, \dots, T$, where $U_{jt} = \partial U_t / \partial x_{jt}$ and $U_{zt} = \partial U_t / \partial z_t$. Equation (3e) shows that the marginal utility of income does not change over time. Therefore, the first-order conditions can be rewritten as

$$U_{jt} - p_{jt} U_{zt} \leq 0, \quad (4a)$$

$$\xi^{t-1} U_{zt} = \lambda, \quad (4b)$$

$$x_{jt} \geq 0, \quad (4c)$$

$$x_{jt} [U_{jt} - p_{jt} U_{zt}] = 0, \quad (4d)$$

$$y_{t-1} - y_t = \mathbf{p}_t' \mathbf{x}_t + z_t, \quad (4e)$$

for $j = 1, \dots, M$ and $t = 1, \dots, T$. Following PKH, we assume that $\partial U_{zt} / \partial \varepsilon_t = 0$, $\partial U_{jt} / \partial \varepsilon_{kt} = 0 \forall k \neq j$, and $\partial U_{jt} / \partial \varepsilon_{jt} > 0 \forall j$ for all period t . Consequently, these first-order conditions can again be rewritten as

$$\varepsilon_{jt} \leq g_{jt}(\mathbf{x}_t, \mathbf{p}_t, \mathbf{Q}_t, z_t), \quad (5a)$$

$$x_{jt} \geq 0, \quad (5b)$$

$$x_{jt} \left[g_{jt}(\mathbf{x}_t, \mathbf{p}_t, \mathbf{Q}_t, z_t) \right] = 0, \quad (5c)$$

$$\lambda = \xi^{t-1} U_{zt}, \quad (5d)$$

$$y_{t-1} - y_t = \mathbf{p}_t' \mathbf{x}_t + z_t, \quad (5e)$$

for $j = 1, \dots, M$ and $t = 1, \dots, T$, where g_{jt} is the solution to $U_{jt}(\mathbf{x}_t, \mathbf{Q}_t, z_t, g_{jt}) = p_{jt} \cdot U_{zt}(\mathbf{x}_t, \mathbf{Q}_t, z_t, g_{jt})$. In this context, an individual who chooses to consume only the first k goods ($x_{jt} > 0$ $j = 1, \dots, k$, $x_{jt} = 0$ $j = k + 1, \dots, M$) in period t contributes the following to the likelihood function:

$$l = \prod_{t=1}^T l_t \quad (6)$$

$$= \prod_{t=1}^T \left[\int_{-\infty}^{g_{k+1}} \cdots \int_{-\infty}^{g_M} f_{\varepsilon}(g_{1t}, \dots, g_{kt}, \varepsilon_{(k+1)t}, \dots, \varepsilon_{Mt}) \times \text{abs} |J_{kt}| d\varepsilon_{Mt} \cdots d\varepsilon_{(k+1)t} \right],$$

where f_{ε} is the joint density function, J_{kt} is the Jacobian for the transformation from ε_t to $(x_{1t}, \dots, x_{kt}, \varepsilon_{(k+1)t}, \dots, \varepsilon_{Mt})'$.

Preference Specification

The empirical application employs the following additively separable utility function suggested by vHPP:

$$U_t = \sum_j^M \left[\Psi(\varepsilon_{jt}) \ln \left(\phi(\mathbf{q}_{jt})x_{jt} + \theta_{jt} \right) \right] + \frac{1}{\rho} z_t^\rho \quad (7)$$

$$\ln \Psi(\varepsilon_{jt}) = \delta' \mathbf{s}_{jt} + \varepsilon_{jt}$$

$$\ln \phi(\mathbf{q}_{jt}) = \gamma' \mathbf{q}_{jt}$$

where \mathbf{s}_{jt} is a vector of individual, site, or time specific variables, and where $\beta = (\delta, \gamma, \rho^*, \theta^*, \mu^*)$ are estimated parameters that may vary randomly across individuals in the population. Note that this specification introduces quality through the simple repackaging of the parameters ϕ (Grilliches 1964), and that weak complementarity is satisfied for all parameter values ($\partial U_t / \partial q_{jt} = 0$ if $x_{jt} = 0, \forall j$, see Mäler 1974). Thus, the first-order conditions can be rewritten as

$$\varepsilon_{jt} \leq g_{jt}(\mathbf{x}_t, \mathbf{p}_t, \mathbf{Q}_t, z_t), \quad (8a)$$

$$x_{jt} \geq 0, \quad (8b)$$

$$x_{jt} \left[g_{jt}(\mathbf{x}_t, \mathbf{p}_t, \mathbf{Q}_t, z_t) \right] = 0, \quad (8c)$$

$$\lambda = \xi^{t-1} \cdot z_t^{\rho-1}, \quad (8d)$$

$$y_{t-1} - y_t = \mathbf{p}_t' \mathbf{x}_t + z_t, \quad (8e)$$

where

$$g_{jt} = -\delta' \mathbf{s}_{jt} + \ln p_{jt} - \gamma' \mathbf{q}_{jt} + \ln \left(\phi(\mathbf{q}_{jt})x_{jt} + \theta_{jt} \right) - (1 - \rho) \ln z_t \quad (9a)$$

$$= -\delta' \mathbf{s}_{jt} + \ln p_{jt} - \gamma' \mathbf{q}_{jt} + \ln \left(\phi(\mathbf{q}_{jt})x_{jt} + \theta_{jt} \right) + \ln \lambda + (1 - t) \ln \xi. \quad (9b)$$

Substituting (8d) into (8e) and summing over all periods, we obtain the closed-form solution of the Lagrange multiplier (λ) and the Hicksian composite good (z_t)¹:

$$\sum_{t=1}^T (y_{t-1} - y_t - \mathbf{p}_t' \mathbf{x}_t) = \sum_{t=1}^T (\lambda \cdot \xi^{1-t})^{\frac{1}{\rho-1}}, \quad (10a)$$

$$y - \sum_{t=1}^T \mathbf{p}_t' \mathbf{x}_t = \lambda^{\frac{1}{\rho-1}} \cdot \sum_{t=1}^T \xi^{\frac{1-t}{\rho-1}}, \quad (10b)$$

$$\lambda = \left[\frac{y - \sum_{t=1}^T \mathbf{p}_t' \mathbf{x}_t}{\sum_{t=1}^T \xi^{\frac{1-t}{\rho-1}}} \right]^{\rho-1}, \quad (10c)$$

$$z_t = \left(y - \sum_{t=1}^T \mathbf{p}_t' \mathbf{x}_t \right) \cdot \left[\frac{\xi^{\frac{1-t}{\rho-1}}}{\sum_{t=1}^T \xi^{\frac{1-t}{\rho-1}}} \right]. \quad (10d)$$

Substituting (10d) into (9a), or substituting (10c) into (9b), we can rewrite equation (9)

as

$$\begin{aligned} g_{jt} = & -\delta' \mathbf{s}_{jt} + \ln p_{jt} - \gamma' \mathbf{q}_{jt} + \ln (\phi(\mathbf{q}_{jt})x_{jt} + \theta_{jt}) + (1-t) \ln \xi \\ & - (1-\rho) \left[\ln \left(y - \sum_{t=1}^T \mathbf{p}_t' \mathbf{x}_t \right) - \ln \left(\sum_{t=1}^T \xi^{\frac{1-t}{\rho-1}} \right) \right]. \end{aligned} \quad (11)$$

Assume that ε_{jt} is an independent and identically distributed draw from the type I extreme value distribution, for all j and t . Then, the likelihood function is as follows:

$$l_t = |\mathbf{J}_t| \prod_j \left(\left[\frac{1}{\mu} \exp \left(\frac{-g_{jt}}{\mu} \right) \right]^{1[x_{jt}>0]} G(g_{jt}/\mu) \right), \quad (12)$$

where $1[x_{jt} > 0]$ is an indicator function that is equal to 1 if x_{jt} is strictly positive and equal to 0 otherwise.

Heterogeneity and Serial Correlation

Unobserved heterogeneity can be analyzed in the dynamic Kuhn-Tucker model by considering random parameters. We assume that $\beta = (\delta, \gamma, \rho^*, \theta^*, \mu^*)$ has a multivariate normal distribution with mean \mathbf{b} and variance-covariance matrix Σ_β . The likelihood function for the random parameters no longer has a closed form. Therefore, the estimation of the random parameters requires simulation. As vHPP discussed, estimating the random parameters using the method of simulated maximum likelihood is computationally difficult because the determinant of the Jacobian must be recalculated at each simulation for each observation. Recently, von Haefen and Phaneuf (2005) proposed a Bayesian estimation approach using MCMC methods to solving the static Kuhn-Tucker model when the Jacobian has random parameters. This approach can be easily extended to the case of a dynamic model. The estimation algorithm for a dynamic model in this Bayesian framework is identical to that for the static model proposed by von Haefen and Phaneuf (2005), except for the likelihood function. The specification can be generalized as follows to allow for serial correlation among error components. Assume that the random parameters vary over individuals but are constant across periods for each person. Then, the likelihood function for period t , $l_t(\mathbf{x})$, exhibits serial correlation ($\text{Cov}(l_t(\mathbf{x}), l_{t+1}(\mathbf{x})) \neq 0$), even if the ε_{jt} are not serially correlated ($\text{Cov}(\varepsilon_{jt}, \varepsilon_{jt+1}) = 0$).

As proposed by von Haefen and Phaneuf (2005), we assume the following diffuse priors:

$$\begin{aligned} \mathbf{b} &\sim N(\beta^0, \eta \mathbf{I}_k), \\ \Sigma_\beta &\sim IW(k, \mathbf{I}_k), \end{aligned} \tag{13}$$

where $N(\cdot)$ and $IW(\cdot)$ are the multivariate normal and inverted Wishart distributions, β^0 is a vector of parameters that are estimated by maximum likelihood with fixed parameters, η is a scale that is chosen ($\eta \rightarrow \infty$), k is the dimension of \mathbf{b} , and \mathbf{I}_k is a k -dimensional identity matrix. Then, the conditional posteriors are as follows (where the subscript $n = 1, \dots, N$ is added to index the individuals in the sample):

$$\begin{aligned}
f(\beta_n | \mathbf{b}, \Sigma_\beta, \mathbf{x}_n) &\propto L(\mathbf{x}_n | \beta_n) \phi(\beta_n | \mathbf{b}, \Sigma_\beta), \\
f(\mathbf{b} | \beta_n, \Sigma_\beta) &\propto N(\bar{\beta}_n, \Sigma_\beta / N), \\
f(\Sigma_\beta | \mathbf{b}, \beta_n) &\propto IW\left(K + N, \frac{K\mathbf{I} + N\bar{\mathbf{S}}}{K + N}\right),
\end{aligned} \tag{14}$$

where ϕ is the normal density function, $L(\mathbf{x}_n | \beta_n) = \prod_t l_t(\mathbf{x}_n | \beta_n)$, $\bar{\beta} = \sum_n \beta_n / N$, and $\bar{\mathbf{S}} = \sum_n (\beta_n - \mathbf{b})(\beta_n - \mathbf{b})' / N$. Note that β_n is constant across periods; therefore, this specification can be used to deal with the serial correlation of the likelihood function. These posteriors can be simulated using the Gibbs sampler (Chib and Greenberg 1995; Train 2003). Iteration i of the Gibbs sampler consists of the following steps:

1. Draw \mathbf{b}^i from $N(\bar{\beta}_n^{i-1}, \Sigma_\beta^{i-1} / N)$.
2. Draw Σ_β^i from $IW\left(K + N, (K\mathbf{I} + N\bar{\mathbf{S}}^{i-1}) / (K + N)\right)$.
3. Draw β_n using the Metropolis-Hastings algorithm.
4. Iterate.

After enough iterations of the burn-in process, this algorithm generates random draws from the posterior distributions.

3 WELFARE ESTIMATION

Compensating Variation with Intertemporal Substitution

Let $V(\mathbf{p}, y, \mathbf{Q}, \varepsilon)$ be the solution of the dynamic utility maximization problem, where $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_T)$, $\mathbf{Q} = (\mathbf{Q}_1, \dots, \mathbf{Q}_T)$, and $\varepsilon = (\varepsilon_1, \dots, \varepsilon_T)$. The compensating variation (*CV*) associated with the change in the price and quality from $(\mathbf{p}^0, \mathbf{Q}^0)$ to $(\mathbf{p}^1, \mathbf{Q}^1)$ is defined as

$$V(\mathbf{p}^0, y, \mathbf{Q}^0, \varepsilon) = V(\mathbf{p}^1, y - CV, \mathbf{Q}^1, \varepsilon). \quad (15)$$

Unless preferences are homothetic or quasi-linear in income, no closed-form solution for *CV* exists. Thus, an iterative search procedure is required. vHPP proposed a numerical bisection algorithm that numerically solves the Kuhn-Tucker conditions for the optimal consumption levels. While vHPP proposed nesting the bisection algorithm when solving for *CV*, von Haefen (2004) recently developed a more efficient numerical algorithm that relies on expenditure functions. These studies investigated the static maximization problem; however, the bisection algorithm can be applied to dynamic allocation models as well. As discussed by vHPP, the assumption of additive separability can simplify the process of solving the consumer's problem. From the first-order conditions for our preference specification, the optimal consumption levels can be calculated as

$$x_{jt} \left(\frac{\Psi(\varepsilon_{jt}) \cdot \xi^{t-1}}{p_{jt} \cdot \lambda} - \frac{\theta_{jt}}{\phi(\mathbf{q}_{jt})} - x_{jt} \right) = 0, x_{jt} \geq 0, \quad (16a)$$

$$\lambda = \left[\frac{y - \sum_{t=1}^T \mathbf{p}_t' \mathbf{x}_t}{\sum_{t=1}^T \xi^{\frac{1-t}{\rho-1}}} \right]^{\rho-1}. \quad (16b)$$

These equations suggest that if the analyst knew the optimal value of λ , equation (16a) could be used to solve for each x_{jt} . Therefore, solving the consumer's problem reduces to solving for the optimal value of λ . In particular, it is possible to extend vHPP's numerical bisection algorithm to the following dynamic allocation model:

1. At iteration i , set $\lambda_a^i = (\lambda_l^{i-1} + \lambda_u^{i-1})/2$. To initialize the algorithm, set $\lambda_l^0 = 0$ and $\lambda_u^0 = y^{\rho-1}$.
2. Conditional on λ_a^i , solve for $\mathbf{X}^i = [x_{jt}^i]$ using the Kuhn-Tucker first-order condition (16a).
3. Conditional on $\mathbf{X}^i = [x_{jt}^i]$, calculate λ^i using equation (16b).
4. If $\lambda^i > \lambda_a^i$, set $\lambda_l^i = \lambda_a^i$ and $\lambda_u^i = \lambda_u^{i-1}$. Otherwise, set $\lambda_l^i = \lambda_l^{i-1}$ and $\lambda_u^i = \lambda_a^i$.
5. Iterate until $abs(\lambda_l^i - \lambda_u^i) \leq c$, where c is arbitrarily small.
6. Calculate $\mathbf{z} = (z_1, \dots, z_T)'$ using equation (8d), and calculate $V(\mathbf{p}, y, \mathbf{Q}, \varepsilon)$ conditional on $\mathbf{X} = [x_{jt}]$ and $\mathbf{z} = (z_1, \dots, z_T)'$ using equation (7).

This procedure yields the optimal consumption $(\mathbf{X}^*, \mathbf{z}^*)$ and optimal utility level V , which are evaluated conditional on $(\mathbf{p}, y, \mathbf{Q}, \varepsilon)$. A similar iterative algorithm can be used to solve for compensating variation. Assuming that utility is strictly concave, the following algorithm finds the values of x and z that minimize expenditure:

1. At iteration i , set $\lambda_a^i = (\lambda_l^{i-1} + \lambda_u^{i-1})/2$. To initialize the algorithm, set $\lambda_l^0 = 0$ and $\lambda_u^0 = y^{\rho-1}$.
2. Conditional on λ_a^i , solve for $\mathbf{X}^i = [x_{jt}^i]$ using the Kuhn-Tucker first-order condition (16a).

3. Conditional on $\mathbf{X}^i = [x_{jt}^i]$, calculate z_t^i using (10d), and $V^i = \sum_{t=1}^T \xi^{t-1} U_t(\mathbf{x}_t^i, \mathbf{Q}_t, z_t^i, \varepsilon_t)$ using (7).
4. If $V^i > V^0$, set $\lambda_l^i = \lambda_a^i$ and $\lambda_u^i = \lambda_u^{i-1}$. Otherwise, set $\lambda_l^i = \lambda_l^{i-1}$ and $\lambda_u^i = \lambda_a^i$.
5. Iterate until $abs(\lambda_l^i - \lambda_u^i) \leq c$, where c is arbitrarily small.
6. Calculate $CV = y - \sum_{t=1}^T (\mathbf{p}'_t \mathbf{x}_t + z_t)$ conditional on $\mathbf{X} = [x_{jt}]$ and $\mathbf{z} = (z_1, \dots, z_T)'$.

Note that the compensating variation $CV(\mathbf{p}^0, \mathbf{Q}^0, \mathbf{p}^1, \mathbf{Q}^1, y, \varepsilon)$ has random components that are unknown to the analyst. For this reason, it is impossible to estimate the exact value of CV . However, as shown by vHPP, the expectation of the compensating variation $E[CV]$ can be estimated using the first-order conditions and a simulation technique for the unobserved components ε . If good j is consumed in strictly a positive quantity in period t ($x_{jt} > 0$), the first-order condition of (5a) implies that $\varepsilon_{jt} = g_{jt}(\mathbf{x}_t, \mathbf{p}_t, \mathbf{Q}_t, z_t)$. Otherwise, given the independent identically distributed (iid) type I extreme value distribution assumption, $\varepsilon_{jt} < g_{jt}(\mathbf{x}_t, \mathbf{p}_t, \mathbf{Q}_t, z_t)$ can be simulated. Thus, the unobserved components ε can be constructed as

$$\begin{aligned} \varepsilon_{jt} &= g_{jt}(\mathbf{x}_t, \mathbf{p}_t, \mathbf{Q}_t, z_t) \quad \text{for } x_{jt} > 0, \\ \varepsilon_{jt} &= -\ln(-\ln(G(g_{jt}/\mu)U))\mu \quad \text{for } x_{jt} = 0, \end{aligned} \tag{17}$$

where G is the iid type I extreme value distribution with a scale of 1, and where U is a random draw from the uniform distribution.

Compensating Variation without Intertemporal Substitution

For comparison purposes, consider compensating variation in the absence of intertemporal substitution. As mentioned above, our proposed model allows us to deal with intertemporal substitution through the consumer's intertemporal budget constraint. If the

individual's budget in each period ($y_{t-1} - y_t \equiv b_t$) is given, then demand is not affected by changes in the prices or qualities in other periods, even if the demand functions exhibit income effects. Consider the following dynamic utility maximization problem:

$$\begin{aligned}
V &= \underset{\mathbf{x}_t, z_t}{\text{Max}} \sum_{t=1}^T \xi^{t-1} U_t(\mathbf{x}_t, \mathbf{Q}_t, z_t, \varepsilon_t) \\
\text{s.t. } & b_t = \mathbf{p}_t' \mathbf{x}_t + z_t, \\
& \sum_{t=1}^T b_t = y, \\
& x_{jt} \geq 0.
\end{aligned} \tag{18}$$

Note that the budget in each period (b_t) is given and that the consumer cannot control budget allocation across periods in this model. In this context, the Kuhn-Tucker first-order conditions in (5) can be rewritten as

$$\begin{aligned}
\varepsilon_{jt} &\leq g_{jt}(\mathbf{x}_t, \mathbf{p}_t, \mathbf{Q}_t, b_t - \mathbf{p}_t' \mathbf{x}_t), \\
x_{jt} &\geq 0, \\
x_{jt} \left[g_{jt}(\mathbf{x}_t, \mathbf{p}_t, \mathbf{Q}_t, b_t - \mathbf{p}_t' \mathbf{x}_t) \right] &= 0,
\end{aligned} \tag{19}$$

where

$$g_{jt} = -\delta' \mathbf{s}_{jt} + \ln p_{jt} - \gamma' \mathbf{q}_{jt} + \ln(\phi(\mathbf{q}_{jt})x_{jt} + \theta_{jt}) - (1 - \rho) \ln(b_t - \mathbf{p}_t' \mathbf{x}_t) \tag{20}$$

for the proposed preference specification. Note that (19) is similar to conditions of the static Kuhn-Tucker model. The optimal consumption level in this case is given by

$$x_{jt} \left(\frac{\Psi(\varepsilon_{jt})}{p_{jt} \cdot z_t^{\rho-1}} - \frac{\theta_{jt}}{\phi(\mathbf{q}_{jt})} - x_{jt} \right) = 0, x_{jt} \geq 0, z_t = b_t - \mathbf{p}_t' \mathbf{x}_t, \quad (21)$$

The demand function in period t is independent of price or quality changes in other periods; therefore, there is no intertemporal substitution. (19) suggests that this dynamic model without intertemporal budget allocation is essentially equivalent to a static model in its first-order conditions. Thus, compensating variation in the absence of intertemporal substitution can be calculated by means of vHPP's numerical bisection algorithm, which was developed for the static model.

4 APPLICATION

Data

To illustrate our proposed dynamic Kuhn-Tucker approach, we discuss an application of our model to beach recreation data for Southern California. Southern California beaches attract 175 million visitors each year, and these visitors spend more than \$1.5 billion during their visits (California EPA 2001). A panel of Southern Californian respondents was recruited between November 1999 and January 2000. They were asked to keep a record of all their trips to beaches in Southern California between December 1999 and November 2000. Six bimonthly diary surveys (waves 1-6) were conducted during this time. Each wave could be regarded as a single period of the dynamic Kuhn-Tucker model; therefore, our data spanned six periods ($t = \{1, 2, \dots, 6\}$). At the end of each wave, the panel members were interviewed by phone. We used only the data for the respondents who answered all six surveys because we wanted to focus on the dynamic behavior of beach users. The final sample consisted of 1,407 trips taken by 250 respondents who supplied sufficient data for our analysis. To be included, a given trip had to have a valid destination, and the re-

spondent who took the trip had to have supplied all of the demographic variables that were included in the model and were needed to calculate travel costs. Multiple beach destination trips or multi-day trips were excluded from the analysis. Table 2 describes the trip distribution of our data across all waves. Note that, while 23% of the respondents did not visit a beach, 50% visited more than one site.

TABLE 1. Diary Surveys

TABLE 2. Distribution of Trips

The number of beach sites in our data was 52 ($j = \{1, 2, \dots, 52\}$). The attributes of the beaches and other selected variables are described in Table 3. The travel cost was modeled as a function of reported respondent income and the estimated vehicle operation cost (\$0.145/mile), travel time, and distance between the respondent's residence and each beach in the choice set. One-way travel distance and travel time between each respondent address and the corresponding beach address were calculated using the computer program PC-Miler. This cost was calculated as follows:

$$\text{cost} = 2 * [\text{one way travel dist} * 0.145 + (\text{one way travel time}) * (0.5 * \text{hourly wage})].$$

Beach water and sand quality data were obtained from the Southern California non-profit group Heal the Bay. These data contained ratings for beach water quality for dry days, on a scale of A+ to F, for many monitoring stations throughout Southern California and for a selection of dates between June 1998 and April 2001.

TABLE 3. Variable List

Results

Table 4 provides the parameters that we estimated using the maximum likelihood method and the Bayesian framework for the dynamic Kuhn-Tucker model with a discount factor

assumed to be 1. For the Bayesian model, the parameters were estimated using 30,000 iterations of the Gibbs sampler. The first 10,000 iterations were discarded as burn-in, and every tenth draw thereafter was retained. As indicated in the table, all of the estimated parameters for both models have the expected signs and are statistically significant at the 1% level, except for the means of the psi function in the Bayesian model. The variables “condoshotels,” “bikepath,” “sandy,” “tidepools,” and “water” had positive effects on utility, while “rocky” and “oilpumps” had negative effects. The coefficients on the dummy variables for each wave may reflect either seasonal variation in preferences or variation in omitted beach attributes, such as water temperature.

TABLE 4. Estimated Parameters

Welfare Estimation

We analyzed two scenarios designed to capture spatial and intertemporal substitution for beach recreation: (A) closure of the Huntington City, Huntington State, and Newport beaches in period 4 and (B) closure of all the beaches in period 4. The first policy scenario concerns a possible closure of the Huntington beaches caused by an oil spill or deterioration in water quality. For example, on February 7, 1990, the tank vessel *American Trader* ran aground and spilled 416,598 gallons of crude oil into the Pacific Ocean offshore of Huntington Beach. Approximately 14 miles of the Newport and Huntington beaches were closed for up to 34 days. For the details of the *American Trader* case, see Chapman and Hanemann (2001). Huntington Beach was also closed during July and August 1999 due to elevated levels of total coliform, fecal coliform, and enterococci bacteria in the water. In our sample, 21 visitors took a total of 88 trips to Huntington City Beach; 27 visitors took a total of 72 trips to Huntington State Beach; and 46 visitors took a total of 171 trips to Newport Beach. Note that the beaches closed under this scenario are only two sites out of 52 beaches that were available in a given period. Therefore, under such a scenario, both

intertemporal substitution and spatial substitution for beach recreation may be observed. In contrast, the second scenario is relevant exclusively to intertemporal substitution. Under this scenario, there can be no spatial substitution because all beaches are closed during a specific period. Therefore, individuals cannot visit any sites during this time.

Let $E[x_t] = E[\sum_j x_{jt}]$ represent the expected number of trips in period t , $E[x] = E[\sum_t \sum_j x_{jt}]$ denote the expected number of trips in all periods, x_t^{close} be the number of trips in period t when the beaches are assumed to be closed in period 4, and x_t^{baseline} be the baseline number of trips in period t . Then the substitution effects for the trips under each scenario can be classified into the following four cases:

A1: Spatial and Intertemporal Substitution

$$\begin{aligned} E[x_t^{\text{close}}] &\geq E[x_t^{\text{baseline}}], \forall t \neq 4, \\ E[x_4^{\text{close}}] &\leq E[x_4^{\text{baseline}}] \end{aligned} \tag{22}$$

A2: Spatial Substitution Only

$$\begin{aligned} E[x_t^{\text{close}}] &= E[x_t^{\text{baseline}}], \forall t \neq 4, \\ E[x_4^{\text{close}}] &\leq E[x_4^{\text{baseline}}] \end{aligned} \tag{23}$$

B1: Intertemporal Substitution Only

$$\begin{aligned} E[x_t^{\text{close}}] &\geq E[x_t^{\text{baseline}}], \forall t \neq 4, \\ E[x_4^{\text{close}}] &= 0 \end{aligned} \tag{24}$$

B2: No Substitution

$$\begin{aligned}
E[x_t^{\text{close}}] &= E[x_t^{\text{baseline}}], \forall t \neq 4, \\
E[x_4^{\text{close}}] &= 0
\end{aligned}
\tag{25}$$

Table 5 reports the predicted trips for the baseline (status quo) and each of the two scenarios. The values for A1 (spatial and intertemporal substitution) were calculated by the means of the first-order conditions of the dynamic Kuhn-Tucker model with intertemporal substitution for scenario A, using (16). Table 5 suggests that (22) holds for the predicted trips. However, the difference between $E[x_t^{\text{baseline}}]$ and $E[x_t^{\text{close}}]$, for $\forall t \neq 4$, is small for both models. Therefore, intertemporal substitution effects under this scenario are relatively small. The number of trips for A2 (spatial substitution only) were calculated by means of the first-order conditions, without intertemporal substitution for scenario A, using (21). We assumed that $b_t = y/T, \forall t$. Note that the number of predicted trips for A2 (spatial substitution only) is similar to that for A1 (spatial and intertemporal substitution). For scenario B, the values for B1 (intertemporal substitution only) were calculated by the means of the first-order conditions of the dynamic Kuhn-Tucker model with intertemporal substitution. Recall that no spatial substitution can occur under scenario B because all beaches are closed in each period. Therefore, individuals cannot visit any sites during the relevant period. Table 5 shows that $E[x_t^{\text{close}}]$, for $\forall t \neq 4$, for B1 (intertemporal substitution only) is larger than $E[x_t^{\text{close}}]$, for $\forall t \neq 4$, for A1 (spatial and intertemporal substitution), as well as $E[x_t^{\text{baseline}}]$. This result suggests that intertemporal substitution effects may be stronger when no spatial substitution is possible.

TABLE 5. Predicted Trips

The compensating variations for the two scenarios are reported in Table 6. Table 6 presents the *CV* per respondent, as defined in (15), and the *CV* per trip (unit value) de-

fined by the ratio between the *CV* per respondent and the change in the total number of trips. As expected, the absolute values of the welfare loss estimated by the dynamic Kuhn-Tucker model with intertemporal substitution are lower than those from the model without intertemporal substitution for both scenarios and under both classical and Bayesian estimations. Under the classical model, an average of \$23.7 per respondent would be have been required to compensate for the closure of all beaches in period 4 (June and July), but the magnitude of this estimated welfare loss rises to \$24.1 if intertemporal substitution is not considered. Under the Bayesian model, the disparity in welfare loss between A1 and A2 (or B1 and B2) is very small but significantly different from zero. Therefore, disregarding intertemporal substitution effects may cause an overstatement of welfare loss, although the difference between the compensating variations of the models with and without intertemporal substitution effects is not relatively large.

TABLE 6. Compensating Variations

5 CONCLUDING COMMENTS

In this paper, we have developed a dynamic Kuhn-Tucker model of recreation demand with intertemporal substitution effects from both theoretical and empirical perspectives. We have extended the traditional, static Kuhn-Tucker model to a dynamic model and developed a methodology for estimating the parameters of the utility function and the compensating variation for this new model. In an empirical application to Southern California beaches, we compared our proposed dynamic model to a model without intertemporal substitution by using both classical and Bayesian frameworks. The empirical results suggest that disregarding intertemporal substitution effects may cause an overstatement of welfare loss. However, the difference in the compensating variations of the dynamic models with and without intertemporal substitution effects was not relatively large for our dataset.

We suggest three additional avenues of investigation via the dynamic Kuhn-Tucker approach. First, stochastic recreation behavior could be investigated. For example, an individual may not know for sure when in the future a beach will close due to an unexpected oil spill or deterioration in water quality. In the discrete-choice literature, stochastic dynamic programming approaches have been proposed for dealing with iterated choice behavior under uncertainty². The stochastic dynamic behavior model could be integrated with the dynamic Kuhn-Tucker approach. Second, the dynamic process of variation in preferences could be investigated. For example, when a familiar site is closed, an individual may visit an unfamiliar site, which may make a favorable impression on the visitor. Our empirical model includes preference variation in the form of dummy variables; however, the dynamic process by which tastes evolve during the season could also be investigated. Finally, the assumption that preferences are additively separable should be relaxed, and the implications of this modification should be examined. Under the assumption of additive separability, the consumer cannot adjust the composite good (z_t) across periods. Although the consumer can allocate his residual income (y_t) across periods and adjust the budget of each period to achieve intertemporal substitution, this assumption may underlie the fact that the intertemporal substitution effects in our analysis were small. Therefore, further conceptual and empirical studies should address problems that integrate non-additively separable utility functions into the dynamic Kuhn-Tucker model.

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Notes

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¹ Equation (10d) suggests that $z_1 = z_2 = \dots = z_T$ when the discount factor $\xi = 1$. This means that the consumer cannot adjust z_t across periods, due to the assumption of additively separable preferences. Although the consumer can allocate y_t across periods and adjust the budget of each period for the purpose of intertemporal substitution, this assumption may be related to the fact that the intertemporal substitution effects were found to be small in our data.

² See, for example, Wolpin (1984, 1987), Pakes (1986), Rust (1987), and Berkovec and Stern (1991). Provencher and Bishop (1997) provided applications to recreation demand.

TABLE 1. Diary Surveys

wave 1 ($t=1$)	December 1999 and January 2000
wave 2 ($t=2$)	February and March 2000
wave 3 ($t=3$)	April and May 2000
wave 4 ($t=4$)	June and July 2000
wave 5 ($t=5$)	August and September 2000
wave 6 ($t=6$)	October and November 2000

TABLE 2. Distribution of Trips

Number of trips taken per respondent	Number of observations		Number of sites visited per respondent		Number of observations
0	57	23%	0	57	23%
1	36	14%	1	69	28%
2	35	14%	2	60	24%
3	20	8%	3	27	11%
4	18	7%	4	21	8%
5	10	4%	5	13	5%
6	12	5%	6	3	1%
7	8	3%	7	0	0%
8	3	1%	8	0	0%
9	6	2%	9	0	0%
10-	45	18%	10-	0	0%
Total	250	100%	Total	250	100%

TABLE 3. Variable List

Variables	Description	Mean	Standard Deviation
$x = \sum_j \sum_t x_{jt}$	number of trips in all waves	5.628	9.513
$x_1 = \sum_j x_{j1}$	number of trips in wave 1	1.056	2.965
$x_2 = \sum_j x_{j2}$	number of trips in wave 2	0.848	2.065
$x_3 = \sum_j x_{j3}$	number of trips in wave 3	0.772	1.745
$x_4 = \sum_j x_{j4}$	number of trips in wave 4	1.508	2.619
$x_5 = \sum_j x_{j5}$	number of trips in wave 5	1.004	2.584
$x_6 = \sum_j x_{j6}$	number of trips in wave 6	0.440	1.445
cost	travel cost	52.867	42.894
income	household income (\$)	62,960	40,093
condoshotels	1/0, 1 = condo(s) or hotel(s) visible from the beach	0.327	0.470
bikepath	1/0, 1 = bike path is available	0.442	0.498
sandy	1/0, 1 = beach is sandy	0.865	0.342
rocky	1/0, 1 = beach is rocky	0.462	0.499
tidepools	1/0, 1 = tide pools at the beach	0.269	0.444
oilpumps	1/0, 1 = onshore oil pumps visible from the beach	0.039	0.193
water	average grade of beach water quality	3.443	1.142
t1	1/0, dummy variable for wave 1	0.167	0.373
t2	1/0, dummy variable for wave 2	0.167	0.373
t3	1/0, dummy variable for wave 3	0.167	0.373
t4	1/0, dummy variable for wave 4	0.167	0.373
t5	1/0, dummy variable for wave 5	0.167	0.373

TABLE 4. Estimated Parameters ^a

		Fixed Parameter		Random Parameter ^b			
				Mean		Std. Dev.	
Psi ($\ln \Psi$)	constant	-6.5486	(0.427)	0.4148	(0.533)	1.1109	(0.176)
	t1	0.6801	(0.133)	0.4154	(0.158)	0.8274	(0.095)
	t2	0.5893	(0.135)	0.3066	(0.180)	0.8509	(0.097)
	t3	0.5438	(0.136)	0.3017	(0.137)	0.8454	(0.099)
	t4	0.9783	(0.128)	0.8521	(0.141)	0.8574	(0.106)
	t5	0.6619	(0.133)	0.4920	(0.159)	0.8570	(0.093)
Quality Index ($\ln \phi$)	condoshotels	0.6665	(0.062)	0.7454	(0.110)	0.9495	(0.089)
	bikepath	0.2140	(0.077)	0.4655	(0.121)	1.0391	(0.099)
	sandy	1.3871	(0.211)	1.8637	(0.367)	1.3171	(0.212)
	rocky	-0.2225	(0.083)	-0.4093	(0.128)	0.8528	(0.082)
	tidepools	0.6269	(0.103)	0.4118	(0.155)	1.0359	(0.121)
	oilpumps	-3.0687	(0.261)	-2.9019	(0.642)	1.8959	(0.352)
	water	0.0910	(0.031)	0.2053	(0.051)	0.4656	(0.030)
Additional Parameters	ρ^*	-0.3823	(0.063)	-2.7885	(0.547)	0.9170	(0.141)
	θ^*	2.9088	(0.261)	5.5517	(0.462)	1.2240	(0.208)
	μ^*	-0.1960	(0.025)	-0.7049	(0.056)	0.4902	(0.032)
LogL		-4790.025					

^a For the classical estimates, robust standard errors are reported in parentheses. For the Bayesian estimates, posterior standard errors are reported in parentheses.

^b The posterior means and standard errors were generated using 30,000 Gibbs sampler iterations. The simulations from the first 10,000 iterations were discarded as burn-in, and the simulations from every tenth iteration thereafter were used to construct the reported estimates.

TABLE 5. Predicted Trips ^a

	Classical						
	$E[x_1]$	$E[x_2]$	$E[x_3]$	$E[x_4]$	$E[x_5]$	$E[x_6]$	$E[x]$
Baseline	1.0560	0.8480	0.7720	1.5080	1.0040	0.4400	5.6280
A1: Spatial and Intertemporal Substitution	1.0562	0.8482	0.7722	1.2162	1.0044	0.4401	5.3373
A2: Spatial Substitution Only	1.0560	0.8480	0.7720	1.2173	1.0040	0.4400	5.3373
B1: Intertemporal Substitution Only	1.0574	0.8489	0.7738	0.0000	1.0063	0.4408	4.1273
B2: No Substitution	1.0560	0.8480	0.7720	0.0000	1.0040	0.4400	4.1200
	Bayesian						
	$E[x_1]$	$E[x_2]$	$E[x_3]$	$E[x_4]$	$E[x_5]$	$E[x_6]$	$E[x]$
Baseline	1.0560	0.8480	0.7720	1.5080	1.0040	0.4400	5.6280
A1: Spatial and Intertemporal Substitution	1.0561	0.8481	0.7721	1.2161	1.0041	0.4401	5.3365
A2: Spatial Substitution Only	1.0560	0.8480	0.7720	1.2164	1.0040	0.4400	5.3364
B1: Intertemporal Substitution Only	1.0567	0.8484	0.7726	0.0000	1.0046	0.4403	4.1225
B2: No Substitution	1.0560	0.8480	0.7720	0.0000	1.0040	0.4400	4.1200

^a For the classical estimates, predicted demands were calculated using the parameters estimated via the maximum likelihood method. For the Bayesian estimates, predicted demands were calculated using the posterior means that were generated using 30,000 Gibbs sampler iterations. Simulations from the first 10,000 iterations were discarded as burn-in, and every tenth iteration thereafter was used to construct the reported estimates.

TABLE 6. Compensating Variations ^a

	Classical		Bayesian	
	CV / person	CV / trip	CV / person	CV / trip
A1: Spatial and Intertemporal Substitution	-3.3181 (0.158)	11.4151 (0.543)	-2.3896 (0.342)	8.1980 (1.171)
A2: Spatial Substitution Only	-3.3572 (0.160)	11.5480 (0.548)	-2.3950 (0.342)	8.2140 (1.171)
A2 - A1	-0.0390 (0.003)	0.1329 (0.010)	-0.0054 (0.001)	0.0159 (0.004)
B1: Intertemporal Substitution Only	-23.6145 (0.896)	15.7356 (0.595)	-18.8689 (1.803)	12.5332 (1.195)
B2: No Substitution	-24.2689 (0.916)	16.0934 (0.607)	-18.9714 (1.804)	12.5805 (1.197)
B2 - B1	-0.6544 (0.047)	0.3578 (0.026)	-0.1026 (0.016)	0.0473 (0.009)

^a For the classical estimates, standard errors calculated using the procedure reported by Krinsky and Robb (1986) and based on 500 iterations are reported in parentheses. For the Bayesian estimates, posterior standard errors are reported in parentheses.

^b The posterior means and standard errors were generated using 30,000 Gibbs sampler iterations. The simulations from the first 10,000 iterations were discarded as burn-in, and every tenth iteration thereafter was used to construct the reported estimates.