

Environmental Goods and Cost-Share System

Koichi Kuriyama

*Department of Agricultural
Economics, Kyoto University*

1. INTRODUCTION

We think the environment has two characteristics -- non-excludability, and non-rivalry. Samuelson offered pure public goods which the individual consumes in the same quantity (Samuelson [1]). But the environment is not always consumed so. Some consumers living near the forest get more the natural environment than

other consumers living in the city. The environmental consumption depends upon the area where the consumer lives. We define Environmental Goods which is non-excludability and non-rivalry, but is not consumed in the same quantity.

This paper is organized as follows:

Section 2 presents the definition of Environmental Goods and the basic model. We discuss the relation between public goods and Environmental Goods.

The center of this paper is Section 3 : it presents the relation between local public goods and Environmental Goods. Environmental Goods is a kind of local public goods, because environmental consumption depends upon the local government. But it is not club goods, because it has non-excludability and non-rivalry.

Environmental Goods need not Tiebout hypothesis. When we move, we consider a house rent, distance of the place of work, station, school, and so on

rather than the natural environment, such as the forest resource and water resource. We do not "vote with the feet" for Environmental Goods. We act at random for the environment. We call this behavior Entropy hypothesis. This hypothesis simplify cost-share system of Environmental Goods. Under this hypothesis, optimal cost-share ratio is equal to the environmental ratio.

Section 4 is final remarks. It mentions the question of assessment of Environmental Goods.

2. DEFINITION OF ENVIRONMENTAL GOODS

2.1 The Basic Model

In this section, we first describe the structure of the basic model and then the condition for Pareto optimality. we use the following notation

x_i^j =the amount of private goods consumed by

individual i in the local government j

$(i=1, \dots, N(j)) \quad (j=1, \dots, m)$

z_i^j = the amount of Environmental Goods consumed

by individual i in the local government j

$X = \sum \sum x_i^j$ total emission of private goods

j, i

Z = total emission of Environmental Goods

$U_i^j(x_i^j, z_i^j)$ = utility function of individual i

in the local government j

$F(X, Z)$ = production set of farm.

$G_i^j(Z)$ = the local characteristic function of

individual i in the local government j

$S(j) = \{1, 2, \dots, N(j)\}$ the set of the individuals

in the local government j .

$T = \{1, 2, \dots, m\}$ the set of local governments.

$N(j)$ = the population of local government j

For example, z is the forest resource.

Environmental Goods is non-excludability and non-rivalry, but it is not consumed in same

quantity. The consumption depends upon the area where the consumer lives. The landscape of the forest is non-excludability and non-rivalry. But the individual living in the forest consumes more the landscape than other individual living in the city. The landscape (z_i^j) of the individual i in j depends upon the forest resource (Z) and the local characteristic (G_i^j).

We utilize the model of Baumol and Oates [2]. Pareto optimal allocation is the solution of following problem:

(1)

$$\text{Max } U_1^1(x_1^1, z_1^1)$$

Subject to

$$U_i^j(x_i^j, z_i^j) \geq U_i^{*j} \quad (i=1, \dots, N(j))$$

$$F(X, Z) \leq 0 \quad (j=1, \dots, m)$$

$$\sum_{j=1}^m \sum_{i=1}^{N(j)} x_i^j \leq X$$

$$z_i^j \leq G_i^j(Z)$$

$$\text{all } x_i^j \geq 0, \quad z_i^j \geq 0, \quad Z \geq 0$$

We assume this model satisfies concave convexity. Because of this assumption, we can use the Kuhn-Tucker theorem. The Lagrangian is following:

(2)

$$L \equiv \sum_{j=1} \sum \lambda_{i^j} [U_{i^j}(\cdot) - U^{*i^j}] - \mu F(\cdot) + \omega (X - \sum_{j=1} \sum x_{i^j})$$

$$+ \sum_{j=1} \sum \xi_{i^j} [G_{i^j}(Z) - z_{i^j}]$$

$(\lambda_{i^j}, \mu, \omega, \xi_{i^j} : \text{Lagrange multipliers})$

Pareto optimality for Environmental Goods requires the following first-order condition:

(3)

$$\lambda_{i^j} \frac{\partial U_{i^j}}{\partial x_{i^j}} - \omega \leq 0 \qquad x_{i^j} (\lambda_{i^j} \frac{\partial U_{i^j}}{\partial x_{i^j}} - \omega) = 0$$

$$-\mu \frac{\partial F}{\partial X} + \omega \leq 0 \qquad X (-\mu \frac{\partial F}{\partial X} + \omega) = 0$$

$$\lambda_{i^j} \frac{\partial U_{i^j}}{\partial z_{i^j}} - \xi_{i^j} \leq 0$$

$$z_{i^j} (\lambda_{i^j} \frac{\partial U_{i^j}}{\partial z_{i^j}} - \xi_{i^j}) = 0$$

$$-\mu \frac{\partial F}{\partial Z} + \sum_{j=1}^m \sum_{i=1}^n \xi_{i^j} \frac{\partial G_{i^j}}{\partial Z} \leq 0$$

$$Z (-\mu \frac{\partial F}{\partial Z} + \sum_{j=1}^m \sum_{i=1}^n \xi_{i^j} \frac{\partial G_{i^j}}{\partial Z}) = 0$$

Eliminating the Lagrange multipliers, we can get the marginal condition:

(4)

$$\sum_{j=1}^m \sum_{i=1}^n \frac{\frac{\partial U_{i^j}}{\partial z_{i^j}} \frac{\partial G_{i^j}}{\partial Z}}{\frac{\partial U_{i^j}}{\partial z_{i^j}} \frac{\partial F}{\partial Z}} = \frac{\frac{\partial F}{\partial Z}}{\frac{\partial X}{\partial Z}}$$

2.2 Environmental Goods and Public Goods

In this basic model, Environmental Goods is a kind of pure public goods. We consider the function $V_{i^j}(x_{i^j}, Z)$ which is the composite function between the area characteristic func-

tion $G_i^j(Z)$ and U_i^j . Then,

$$V_i^j(x_i^j, Z) = U_i^j(x_i^j, G_i^j(Z))$$

Pareto optimality problem become:

(5)

$$\text{Max } V_1^1(x_1^1, Z)$$

Subject to

$$V_i^j(x_i^j, Z) \geq V_i^{*j}$$

$$F(X, Z) \leq 0$$

$$\sum_j \sum_i x_i^j \leq X$$

$$\text{all } x_i^j \geq 0, Z \geq 0$$

This problem is equal to Samuelson's pure public goods. So Lindahl equilibrium can apply to Environmental goods. The difference between pure public goods and Environmental goods is the local characteristic function (G_i^j). Environmen-

tal goods has more information than pure public goods. In section 3, we utilize this information for cost-share system.

2.3 Environmental Goods and Pigouvian Subsidy.

Then, we consider competitive equilibrium of Environmental Goods. The tax and subsidy is :

$t_i^j(x_i^j)$ = the consumption tax function of the
consumer i in the local government j

$t_P(X)$ = the production subsidy function of the
forest

t_S = the Environmental subsidy ratio of the
forest

p = the price of the private goods

We obtain the Lagrangian:

(6)

consumer: $L_i^j = p X_i^j + t_i^j + \alpha_i^j [U_i^{k,j} - U_i^j(\cdot)]$

forest: $L_f = p X + t_P + t_S Z - \beta F(\cdot)$

The first-order condition:

(7)

$$p + \frac{\partial t_i^j}{\partial x_i^j} - \alpha_i^j \frac{\partial U_i^j}{\partial x_i^j} \geq 0 \quad x_i^j \left(p + \frac{\partial t_i^j}{\partial x_i^j} - \alpha_i^j \frac{\partial U_i^j}{\partial x_i^j} \right) = 0$$

$$p + \frac{\partial t_P}{\partial X} - \beta \frac{\partial F}{\partial X} \leq 0 \quad X \left(p + \frac{\partial t_P}{\partial X} - \beta \frac{\partial F}{\partial X} \right) = 0$$

$$t_s - \beta \frac{\partial F}{\partial Z} \leq 0 \quad Z \left(t_s - \beta \frac{\partial F}{\partial Z} \right) = 0$$

By (3) and (7), one of the Pareto optimal tax/subsidy solution (Pigouvian Tax/Subsidy) is :

(8)

$$t_s = \sum_j \sum_i \lambda_i^j \frac{\partial U_i^j}{\partial z_i^j} \frac{\partial G_i^j}{\partial Z}$$

$$\text{all } i, j \quad \frac{\partial t_i^j}{\partial x_i^j} = 0 \quad , \quad \frac{\partial t_P}{\partial X} = 0$$

Thus Pigouvian Tax/Subsidy system can apply to Environmental Goods. Pigouvian subsidy (t_s) is composed of consumer's lump-sum tax.

However, Pigouvian Tax/Subsidy for Environmental Goods has the equitable problem. Environmental Goods is not consumed in the same quantity. For example, some consumers in the city get less Environmental Goods of the forest than other consumers in the forest. The consumer living in the remote and lonely small island can not watch the landscape of the forest, so he consumes Environmental Goods of the forest in the very small quantity. Nevertheless, can we impose lump-sum tax on the consumers? It is true Pigouvian Tax/Subsidy can apply to Environmental Goods logically, but it can not realize because of ethical problem. We should pursue other cost share system.

3. ENVIRONMENTAL COST-SHARE SYSTEM (ECSS)

3.1 Personalized Tax--Lindahl's Approach

As Environmental Goods is not consumed in same quantity, it is difficult to impose the lump-sum tax on the consumers. We should adopt the personalized tax system. Personalized tax ratio (q_i^j) is imposed on the quantity of Environmental Goods (z_i^j) consumed by the individual i in the local area j . Then, the Lagrangian of market equilibrium become:

(9)

$$\text{consumer: } L_i^j = p X_i^j + \alpha_i^j [U_i^{*j} - U_i^j(\cdot)] + q_i^j z_i^j$$

$$\text{forest: } L_f = p X - \beta F(\cdot) + \sum_j \sum_i q_i^j z_i^j$$

$$(z_i^j = G_i^j(Z))$$

First-order condition is:

(10)

$$p - \alpha_{i^j} - \frac{\partial U_{i^j}}{\partial x_{i^j}} \geq 0$$

$$x_{i^j} (p - \alpha_{i^j} - \frac{\partial U_{i^j}}{\partial x_{i^j}}) = 0$$

$$-\alpha_{i^j} - \frac{\partial U_{i^j}}{\partial z_{i^j}} + q_{i^j} \geq 0$$

$$z_{i^j} (-\alpha_{i^j} - \frac{\partial U_{i^j}}{\partial z_{i^j}} + q_{i^j}) = 0$$

$$p - \beta - \frac{\partial F}{\partial X} \leq 0$$

$$X (p - \beta - \frac{\partial F}{\partial X}) = 0$$

$$-\beta - \frac{\partial F}{\partial Z} + \sum_{j=1}^J q_{i^j} \frac{\partial G_{i^j}}{\partial Z} \leq 0$$

$$Z (-\beta - \frac{\partial F}{\partial Z} + \sum_{j=1}^J q_{i^j} \frac{\partial G_{i^j}}{\partial Z}) = 0$$

Eliminating q_{i^j} from B and D:

(11)

$$-\beta - \frac{\partial F}{\partial Z} + \sum_{j=1}^J \alpha_{i^j} \frac{\partial U_{i^j}}{\partial z_{i^j}} \frac{\partial G_{i^j}}{\partial Z} = 0$$

This satisfies the Pareto optimum marginal condition (3). The cost-share (c_i^j) of the consumer i in the local government j is equal to the personalized tax ($q_i^j z_i^j$):

(12)

$$c_i^j = q_i^j z_i^j = \alpha_i^j \frac{\partial U_i^j}{\partial z_i^j} \cdot z_i^j$$

$$= p \cdot z_i^j \cdot \frac{\partial U_i^j / \partial z_i^j}{\partial U_i^j / \partial x_i^j}$$

$$= p \cdot z_i^j \cdot MRS_{i^j}$$

3.2 Local Environmental Goods (1)

The cost-share of Environmental Goods is depends upon the Marginal Rate of Substitute (MRS_{zx}) and the quantity of Environmental Goods (z_i^j). When we enforce the cost-share system, we must get the information of MRS_{zx} and

z_i^j . However, the consumers do not always report true MRS_{zx} . They can "free-ride".

Therefore we offer the assumption of the consumer's preferences which can decide us the optimal cost-share ratio without the information of MRS_{zx} .

ASSUMPTION 1.

The individuals living in the same local government consume the Environmental Goods in the same quantity. Namely,

$$z_1^1 = z_2^1 = z_3^1 = \dots = z_1^j = \dots = z^j.$$

$$z_i^j = z_1^j = z_j^j = \dots$$

Assumption 1 means that the quantity of Environmental Goods depends upon the region where the individual live.

ASSUMPTION 2.

The local governments have very large

population, and we can consider the local governments have approximate same population (individual $i=1, \dots, N$).

Assumption 2 makes the model simple. We will release this assumption later.

ASSUMPTION 3.

There are the individuals who have the similar utility function.

ASSUMPTION 4. (Entropy-Maximizing Hypothesis)

When the individual moves, he dose not consider Environmental Goods. Thus, the individual acts at random for Environmental Goods.

When we move, we consider a house rent, a distance of the place of work, school, station, and so on rather than the natural environment such as the forest, river, and lake. Assumption 4 is

the opposite of Tiebout hypothesis. In Tiebout hypothesis, the individuals "vote with the feet" for the various public packages, as a result, the local government is composed of the individuals which have the similar preferences for the public goods (Tieboaut[*]). In Assumption 4, the individuals do not "vote with the feet" at all, therefore, the local government is composed of the individuals which have the various preferences for the Environmental Goods. In Tiebout hypothesis, the individual moves in the entropy reducing direction, whereas in assumption 4, the individual moves entropy-increasing direction, and the moving is completed when the entropy is the maximum. We call assumption 4 "Entropy-maximizing Hypothesis". The law of Entropy-increasing is natural law. We consider that the individual follows the natural law, as concern the natural environment.

We now state our main result which can

decide us the optimal cost-share ratio:

PROPOSITION 1

Under the assumptions 1, 2, 3, and 4, the optimal cost-share ratio of the local government j is equal the ratio of the consumption of Environmental Goods per head (Z^j).

Proof

From assumption 1, the cost-share (C^j) of the local government j is :

(13)

$$\begin{aligned}
 C^j &= \sum_{i \in S(j)} c_i^j = \sum_i q_i^j z_i^j = \sum_i q_i^j Z^j \\
 &= p Z^j \sum_i \frac{\partial U_i^j}{\partial z_i^j} \bigg/ \frac{\partial U_i^j}{\partial x_i^j}
 \end{aligned}$$

From assumption 3 and 4, the probability of any MRS_{zx} in each local government is equal, when

the moving of the individuals is completed. Namely, the distribution of MRS_{z_ix} in each local government is equal. And with assumption 2, we can specify the individual's utility function:

(14)

$i=1, \dots, N$

$$\frac{\partial U_i^1}{\partial z_i^1} \bigg/ \frac{\partial U_i^1}{\partial x_i^1} = \frac{\partial U_i^2}{\partial z_i^2} \bigg/ \frac{\partial U_i^2}{\partial x_i^2} = \dots = \frac{\partial U_i^m}{\partial z_i^m} \bigg/ \frac{\partial U_i^m}{\partial x_i^m} \equiv \pi_i$$

$$(MRS_{z_i^1} = MRS_{z_i^2} = \dots = MRS_{z_i^m} \equiv \pi_i)$$

Then all probability of MRS_{z_ix} in each local government is 1/N, and the entropy is (See Shannon [*]) :

(15)

$$E = \sum_{i=1}^N \left(- \frac{1}{N} \log_2 \frac{1}{N} \right) = \log_2 N$$

This entropy is the maximum. From (13) and (14),

the cost-share (C^j) of local government j is :

(16)

$$C^j = \rho Z^j \sum_{i=1}^N \frac{\partial U_i^j}{\partial z_i^j} \bigg/ \frac{\partial U_i^j}{\partial x_i^j}$$

$$= \rho Z^j \sum_i \pi_i = \rho Z^j \Pi \quad (\Pi \equiv \sum_{i=1}^N \pi_i)$$

ρ and Π do not depend on the local government (j), therefore the cost-share ratio of the local government j depends upon only Z^j (the consumption of Environmental Goods in the local government j).

Q. E. D

Proposition 1 makes us avoid worrying the free riding problem. When we decide the cost-share, we have only to estimate the consumption of Environmental Goods.

3.3 Local Environmental Goods (2)

We have accepted assumption 2 (the population of the each local government is equal). This assumption seems unrealistic. Now, we release the assumption 2, and proof the proposition 2:

DEFENITION 1

If the cost-share ratio is equal the ratio of whole consumption of Environmental Goods ($Z^j N(j)$) in the local government, we call this cost-share system "Environmental cost-share system: ECSS".

PROPOSITION 2

Under assumption 1, 3, and 4, Environmental cost-share system (ECSS) is the optimal cost share system.

Proof

and Z_k^j is the amount of Environmental Goods consumed by individuals who have V_k and live in the local government j . For example,

$N(1)=10$:the population of the local government 1

$$\phi_1^1=0.3, \quad \phi_2^1=0.5, \quad \phi_3^1=0.2$$

In this case, the structure of individuals is :

individual 1-3 $MRS_1^1=MRS_2^1=MRS_3^1=V_1$

$$q_1^1=q_2^1=q_3^1=Q_1^1$$

$$z_1^1=z_2^1=z_3^1=Z_1^1$$

individual 4-8 $MRS_4^1=MRS_5^1=MRS_6^1=MRS_7^1=MRS_8^1=V_2$

$$q_4^1=\dots=q_8^1=Q_2^1$$

$$z_4^1=\dots=z_8^1=Z_2^1$$

individual 9-10 $MRS_9^1=MRS_{10}^1=V_3$

$$q_9^1=q_{10}^1=Q_3^1$$

$$z_9^1=z_{10}^1=Z_3^1$$

Now, we reconstruct Proposition 1 with V_k and ϕ_k^j .

We can transfer (13) to :

(18)

$$C^j = \sum_{h=1}^z \phi_h^j \cdot N(j) \cdot Q_h^j \cdot Z_h^j$$

$$= p N(j) \sum_h \phi_h^j \cdot V_h \cdot Z_h^j$$

$$\left(Q_h^j = p \frac{\partial U_h^j}{\partial Z_h^j} \middle/ \frac{\partial U_h^j}{\partial x_h^j} = p V_h \right)$$

From Assumption 1, $Z_1^j = Z_2^j = \dots = Z_L^j \equiv Z^j$. We consider the ratio vector ϕ^j :

$$\phi^j = \begin{pmatrix} \phi_1^j \\ \phi_2^j \\ \vdots \\ \phi_L^j \end{pmatrix}$$

Then, the cost-share is :

(19)

$$C^j = p N(j) \cdot Z^j \cdot \sum_h \phi_h^j \cdot V_h$$

$$= p N(j) \cdot Z^j \phi^j \cdot V$$

And from assumption 4, the ratio of MRS in each local government is equal. ($\phi_k^1 = \phi_k^2 = \phi_k^3 = \dots = \phi_k^m \equiv \Phi_k$)

Therefore,

(20)

$$\forall j \in T, \phi^j = \begin{pmatrix} \phi_1^j \\ \phi_2^j \\ \vdots \\ \phi_\tau^j \end{pmatrix} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_\tau \end{pmatrix} \equiv \Phi$$

$$C^j = p N(j) \cdot Z^j \Phi \cdot V$$

$$\therefore \frac{C^j}{C^k} = \frac{N(j) \cdot Z^j}{N(k) \cdot Z^k}$$

p and $\Phi \cdot V$ do not depend upon j . This means the cost-share ratio of local government j depend

upon only $N(j)Z^j$. Thus, Under assumption 1, 3, and 4, environmental cost-share system (ECSS) is optimum cost-share system.

Q. E. D

3.4 The necessary-sufficient condition

We proof the Proposition 2 with the assumption 1, 3, and 4. But the assumption 1, 3, and 4 are not the necessary-sufficient condition always. We consider the necessary-sufficient condition under the assumption 1 and 3. Under the assumption 1 and 3, the cost-share (C^j) of the local government j is:

(21)

$$C^j = p N(j) \cdot Z^j \cdot \sum_h \phi_h^j \cdot V_h$$

$$= p N(j) \cdot Z^j \phi^j \cdot V$$

Therefore, the necessary-sufficient condition for the environmental cost-share system (ECSS) is :

(22)

$$\forall_{j \in T}, \quad \phi^j \cdot V = \text{constant}$$

4. Final Remark

The proposition 2 makes our decision for environmental policy easy. When we can estimate the consumption of the environment, and can not the utility function of the consumers, it is difficult to acquire the optimum cost-share. The proposition 2 offers the optimum cost-share ratio without utility function. The proposition 2 needs the assumption 1, 3, and 4. Using the statistics, we can guess whether the assumptions are true or not.

Tiebout hypothesis is the extreme case, and

entropy maximizing hypothesis (assumption 4) is another extreme case. The real case seems to be in between them.

The proposition 2 offers the optimum cost share ratio, but it can not offer the amount of optimum cost-share. In the assessment of the amount of optimum cost, we need the information about the consumers' utility function. But, the proposition 2 offers only the optimum ratio. This is the defect of the environmental cost share system

Under assumption 4, Henry-George Theorem[*] and Hedonic approach [*] are invalid, because they assume Tiebout hypothesis. When we assess the amount of optimum cost, we need other tools.